



Tutorial for Cyber-Physical Systems - Discrete Models

Exercise Sheet 11

The goal of this sheet is to improve your understanding of the different types of LT properties, i.e., invariants, safety properties and liveness properties.

Exercise 1: Closure Properties of LT Properties

The goal of this task is to understand the effect of set operations on liveness and safety properties.

Let P and P' be liveness properties over AP . Prove or disprove the following claims:

- (a) $P \cup P'$ is a liveness property.
- (b) $P \cap P'$ is a liveness property.

Perform the same tasks for safety properties.

Exercise 2: LT Properties

The goal of this task is to learn to identify the different types of LT properties.

Consider the following LT properties with $AP = \{a, b\}$. State for each of the properties which of them are invariants, which are safety properties, which are liveness properties, and which are neither. Justify your answer!

- (i) Always (at any point of time) a or b holds.
- (ii) Either a holds exactly once, or b never holds.
- (iii) If a holds, then b will never hold in the next step.
- (iv) Every time a holds there will be eventually a point of time where b holds.
- (v) The atomic propositions a and b never hold at the same time.
- (vi) If a holds infinitely often, then b holds infinitely often.
- (vii) There are only finitely many points of time where a holds.

Exercise 3: Decomposition

The goal of this exercise is to understand the relation between any LT property and safety and liveness properties, by applying the decomposition theorem.

According to the decomposition theorem, any LT property P can be decomposed into a safety property P_{safe} and a liveness property P_{live} , such that the property P is equal to their intersection, i.e.,

$$P = P_{safe} \cap P_{live} .$$

Apply the construction in the proof of the decomposition theorem to find the decomposition for the following properties with $AP = \{a, b\}$. In particular, for each property, give its closure. Use set notation ($\{A_0A_1A_2 \dots \mid \forall k \exists j \dots\}$) to express P_{safe} and P_{live} .

- (i) Every a is immediately followed by b .
- (ii) The atomic proposition a holds infinitely often.
- (iii) At exactly 3 points of time, a holds.

Hint: Some tasks may require very little work.