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Tutorials for Decision Procedures
Exercise Sheet 3

Exercise 1: The Rule Propagate 4 Points

The rule Propagate is not necessary for the DPLL algorithm. Show this in the following two ways.

(a) Recheck the correctness and termination proofs.

(b) Show that the effect of Propagate can be emulated by the other rules.

Give a reason why the rule is still useful.

Exercise 2: Syntax of FOL 4 Points

The items (a) to (p) below display strings composed of logical symbols (i.e., $\neg$, $\wedge$, $\exists$, ...) and non-logical symbols (i.e., variables, constants, functions, and predicates).

- For each of the non-logical symbols, infer its type and arity from its usage in the strings below, and from our conventions introduced on the slides. Assume that each such symbol has the same type and arity in all of the strings.
- For each of the strings (a) to (p), determine whether it is a *term*, an *atom*, a *literal*, or a *formula*. Note that it may be more than one, or none of the above (if it is syntactically incorrect).

(a) a	(f) $p(x, y)$	(k) $p(a, b) \vee p(b, a)$
(b) $f(a)$	(g) $\neg p(a, b)$	(l) $p \wedge \exists x.\, p(x, x)$
(c) $g(f(a), f)$	(h) $\exists a.\, p(a, b)$	(m) $\neg\exists x.\, p(a, b)$
(d) $p(f(a), x)$	(i) $\exists x.\, p(x, f(a))$	(o) $\neg(\exists x. \vee \forall x.\, p(x, x))$
(e) $g(x, f(x))$	(j) $p(x, p(x, x))$	(p) $\forall x.\, \exists x.\, p(x, x)$

Exercise 3: FOL Satisfiability 4 Points

For each of the following formulae F_i give an interpretation I_i with $I_i \models F_i$.

(a) $F_1 : equals(add(2, 2), 5)$

(b) $F_2 : \forall x.p(x, x)$

(c) $F_3 : \exists y.\forall x.p(x, y)$

(d) $F_4 : \forall x.(p(x, f(x)) \wedge \neg p(f(x), x))$

Is there an interpretation I under which $F_2 \wedge F_3 \wedge F_4$ is true?