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Tutorials for Decision Procedures
Exercise Sheet 4

Exercise 1: Substitutions 4 Points

Compute the result $F_i\sigma_i$ of the following substitutions.

(a) $F_1 : \forall y.\ p(x, y)$ and $\sigma_1 : \{x \mapsto f(x)\}$

(b) $F_2 : \forall y.\ p(x, y)$ and $\sigma_2 : \{x \mapsto a, y \mapsto a\}$

(c) $F_3 : \forall y.\ (p(x, y) \land \exists x.\ p(y, x))$ and $\sigma_3 : \{x \mapsto f(x)\}$

(d) $F_4 : \forall y.\ p(x, y)$ and $\sigma_4 : \{x \mapsto f(y), y \mapsto f(x)\}$

Exercise 2: Semantic Argument – Quantifier Instantiation 3 Points

Consider the following semantic argument "proof" of validity for the formula

$$(\forall x.p(x, x)) \to (\exists x \forall y.p(x, y)).$$

1.	$I \not\models (\forall x.p(x, x)) \to (\exists x \forall y.p(x, y))$	
2.	$I \models \forall x.p(x, x)$	$1, \to$
3.	$I \not\models \exists x \forall y.p(x, y)$	$1, \to$
4.	$I \not\models \forall y.p(y, y)$	$3, \exists\ (x \mapsto y)$
5.	$I \not\models p(a, a)$	$4, \exists\ (y \mapsto a$ fresh$)$
6.	$I \models p(a, a)$	$2, \exists\ (y \mapsto a)$
7.	$I \models \bot$	(contradiction, 5. 6.)

However this formula is not valid. Where is the mistake in the proof? Explain your answer.

Exercise 3: FOL Validity 5 Points

Which of the following formulae is valid? If it is valid, give a proof using the *semantic argument* method; otherwise, give a *falsifying interpretation*.

(a) $(\forall y.p(y, y)) \to \exists x.p(x, a)$

(b) $(\forall x, y.(p(x, y) \lor p(y, x))) \to \forall z.p(z, z)$

(c) $(\forall x, y.(p(x, y) \to p(y, x))) \to \forall z.p(z, z)$

(d) $(\exists x.p(x)) \to \forall y.p(y)$

(e) $\exists x.(p(x) \to \forall y.p(y))$