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Tutorials for Decision Procedures
Exercise Sheet 5

Exercise 1: Prenex Normal Form — 4 Points

Transform the following formulae into prenex normal form.

(a) $\forall x.\ \big(q(x) \vee (\forall x.\ p(x,x))\big)$

(b) $r(x,y) \wedge \forall x.\ \Big(\big(\forall y.\ p(x,y)\big) \rightarrow q(x)\Big)$

(c) $\big(\exists x.\ \forall y.\ p(x,y)\big) \rightarrow \big(\forall y.\ \exists x.\ p(x,y)\big)$

(d) $\neg\Big(\forall x.\ \big(p(x,x) \rightarrow \forall y.\ p(x,y)\big)\Big)$

Exercise 2: Semantic Argument in T_{E} — 4 Points

Are the following Σ_{E}-formulae valid?
Give a semantic argument proof or a counterexample (i.e., a falsifying T_{E}-interpretation).

(a) $f(x,y) = f(y,x) \rightarrow f(a,y) = f(y,a)$

(b) $\forall x.\ \exists y.\ f(x,y) = f(y,x)$

(c) $f(f(a)) = f(a) \wedge f(f(f(a))) = a \rightarrow f(a) = a$

Exercise 3: Arithmetic over Integers and Natural Numbers — 4 Points

Consider the following formula.

$$F : \exists x. \forall y.\ \neg(y + 1 = x)$$

(a) View F as a $\Sigma_{\mathbb{Z}}$-formula, and prove $T_{\mathbb{Z}}$-unsatisfiability of F following the approach in the lecture, i.e., perform the following steps.

   (i) Convert F into an equisatisfiable $\Sigma_{\mathbb{N}}$-formula G.

   (ii) Prove $T_{\mathbb{N}}$-unsatisfiability of G using the semantic argument. You may assume that associativity and commutativity of addition holds.

(b) View F as a $\Sigma_{\mathbb{N}}$-formula, and prove $T_{\mathbb{N}}$-validity of F using the semantic argument.