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Tutorials for Decision Procedures
Exercise Sheet 6

Exercise 1: $T_{\mathbb{N}}$ vs. $T_{\mathbb{Q}}$ vs. $T_{\mathbb{R}}$ 4 Points

Show validity of the following formula in each of the three theories $T_{\mathbb{N}}$, $T_{\mathbb{Q}}$, and $T_{\mathbb{R}}$ by giving semantic argument proofs.

$$F: \neg(1 + 1 = 0)$$

Exercise 2: Semantic Argument in Theories 4 Points

Show validity of the following formulae in the combination of the theories T_{E}, $T_{\mathbb{Q}}$, T_{cons}, and T_{A}. You can use all axioms of these four theories. You can use abbreviations as in the slides or the book for introducing theory axioms.

(a) $f(x + y) \neq f(x) \to y \neq 0$

(b) $a\langle i \triangleleft a[i]\rangle[j] = a[j]$

(c) $\neg\mathsf{atom}(x) \wedge \mathsf{car}(x) = y \wedge \mathsf{cdr}(x) = z \to \mathsf{cons}(y, z) = x$

(d) $\mathsf{cons}(x, y) = \mathsf{cons}(y, z) \to x = y$

Exercise 3: Quantifier Elimination for $T_{\mathbb{Q}}$ 4 Points

Apply quantifier elimination to the following $\Sigma_{\mathbb{Q}}$-formulae. In each case, eliminate all quantifiers.

(a) $\exists y.\ (x = 2y \wedge y < x)$

(b) $\forall y.\ (25 < x + 2y \vee x + 2y < 25)$

(c) $\forall x.\ \exists y.\ (y > x \wedge -y < x)$

(d) $\forall x.\ (x > 0 \leftrightarrow \exists y.\ (x > y \wedge -x < y))$