Jochen Hoenicke
Tanja Schindler

03.12.2019
submit until 10.12.2019, 14:15

Tutorials for Decision Procedures
Exercise Sheet 7

Exercise 1: Quantifier Elimination for $T_{\mathbb{Q}}$: Sufficient Set 4 Points

For $T_{\mathbb{Q}}$ the quantifier elimination algorithm in the lecture examines terms $\frac{s+t}{2}$ for all $s, t \in S$. Suppose we split up S in S_A, S_B, S_C depending on whether the term t comes from a literal of type (A) $x < t$, (B) $t < x$, or (C) $x = t$. Based on this distinction, give a smaller set of terms that still is sufficient.

Exercise 2: Quantifier Elimination for $\widehat{T_{\mathbb{Z}}}$ 4 Points

Apply quantifier elimination to the following $\Sigma_{\mathbb{Z}}$-formulae. In each case, eliminate all quantifiers.

(a) $\exists y.\ (x = 2y \wedge y < x)$

(b) $\forall y.\ (25 < x + 2y \vee x + 2y < 25)$

(c) $\exists x.\ \exists y.\ 3x + y = 6 \wedge y < x \wedge 0 < x \wedge 0 < y$

Exercise 3: Congruence Closure Algorithm for T_{E} 4 Points

Apply the congruence closure algorithm to decide satisfiability for the following Σ_E-formulae. You do not need to display all the intermediate graphs, only the final graph. But please write down all merge steps in the order they are performed.

(a) $f(a) = f(b) \wedge a \neq b$

(b) $f(f(a)) = f(a) \wedge f(f(f(a))) = a \wedge f(a) \neq a$

(c) $f(f(f(a))) = f(a) \wedge f(f(a)) = a \wedge f(a) \neq a$

(d) $f(g(a)) = g(f(a)) \wedge f(g(f(b))) = a \wedge f(b) = a \wedge g(f(a)) \neq a$

Bonus Exercise 4: Implementing Quantifier Elimination for $T_{\mathbb{Q}}$ 4 Bonus Points

Implement the quantifier elimination algorithm for $T_{\mathbb{Q}}$ from the lecture. SMTInterpol can be started with a special -script option giving a different solver file. This way you do not need to take care of parsing and most other technicalities. A template file, which also contains the NNF-conversion and some more hints, and starting instructions are given on the lecture website.