Jochen Hoenicke
Tanja Schindler

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Tutorials for Decision Procedures
Exercise Sheet 8

Exercise 1: Constructing the DAG 2 Points

Describe a procedure that takes the abstract syntax tree of a conjunctive quantifier-free Σ_E-formula and constructs the corresponding initial DAG for the congruence closure algorithm. The procedure should run in linear time in the size of the formula on average. You can assume an $O(1)$ implementation for hash tables.

Exercise 2: Decision Procedure for quantifier-free $T_{\mathsf{cons}} \cup T_E$ 4 Points

Apply the decision procedure for quantifier-free $T_{\mathsf{cons}} \cup T_{\mathsf{E}}$ to decide satisfiability of the following $\Sigma_{\mathsf{cons}} \cup \Sigma_{\mathsf{E}}$-formulae.

(a) $y = cons(cdr(x), car(x)) \wedge x = cons(car(y), cdr(y))$

(b) $y = cons(cdr(x), car(x)) \wedge x = cons(car(y), cdr(y)) \wedge car(x) \neq cdr(x)$

(c) $\neg atom(x) \wedge y = cons(cdr(x), car(x)) \wedge z = cons(cdr(y), car(y)) \wedge z \neq x$

Exercise 3: Satisfying Interpretation for T_{cons} 4 Points

Take the result of applying the congruence closure algorithm on the following formula.

$$y = cons(cdr(x), car(x)) \wedge x = cons(car(y), cdr(y)).$$

Give a satisfying interpretation I.
Under this interpretation, what is the value of the term $cons(x, cons(car(x), car(y)))$?

Exercise 4: Dutertre–de Moura Algorithm 2 Points

Apply the Dutertre–de Moura algorithm to decide the $T_{\mathbb{Q}}$-satisfiability of the following $\Sigma_{\mathbb{Q}}$-formulae. If a formula is satisfiable, give a satisfying $T_{\mathbb{Q}}$-interpretation.

(a) $x_1 + 2x_2 \geq 1 \wedge 2x_1 + x_2 \geq 1 \wedge x_1 + x_2 \leq 1$

(b) $x_1 + 2x_2 \geq 1 \wedge 2x_1 + x_2 \geq 1 \wedge x_1 + x_2 \leq \frac{1}{2}$