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Tutorials for Decision Procedures
Exercise Sheet 9

Exercise 1: DPLL 4 Points

Santa is desperate: Christmas is approaching fast, but the presents are not yet wrapped – the Christmas present wrapping machine stands still – again! The instructions, however, are cryptic:

> Dear user,
>
> the Christmas present wrapping machine has 8 colored levers. In order to start the machine, they must be in the correct positions.
>
> - If the *blue lever* is in the upper position, then the *red lever* must be in the lower position.
> - If the *green lever* is in the upper position, then the *yellow lever* must be in the lower position.
> - However, if the *orange lever* is in the upper position, then both the *blue lever* and the *violet lever* must be so, too.
> - If the *yellow lever* is in the lower position, then either the *orange lever* or the *pink lever* (but not both!) must be in the upper position.
> - If the *pink lever* is not in the upper position, then the *green lever* must be there if the *white lever* is.
> - If the *white lever* is in the upper position, bring the *pink lever* into the lower position.
> - If either the *blue lever* or the *yellow lever* is in the upper position (but not both), then the *violet lever* must also be in the upper position, if the *red lever* is not.
> - If the *blue lever* is in the upper position, then, if the *green lever* is not but the *violet lever* is in the upper position, the *pink lever* must be in the lower position.
> - Finally, if the *orange lever* is in the lower position, then please bring the *white lever* into the upper position.
>
> Once all levers are brought to the correct position, please press the Start button.

Help Santa to get the machine running again (using the DPLL algorithm)!

Exercise 2: Quantifier-free $T_{\mathbb{Q}}$/Dutertre–de Moura Algorithm 4 Points

Santa has come to think that his reindeers should adopt a healthier lifestyle. For this reason he wants to reduce the calories in his cookie recipe. He searches for a good ratio of sugar (s), butter (b) and flour (f). His constraints for the cookies are as follows:

- He wants a recipe for 1 kg of dough.

$$s + b + f = 1000$$

- Overall calories may not be over 4000. Sugar has 4 calories per gram, butter has 7, flour 3.

$$4 \cdot s + 7 \cdot b + 3 \cdot f \leq 4000$$

- Obviously, he cannot add a negative amount of an ingredient.

$$s, b, f \geq 0$$

- He needs at least as much butter as sugar, otherwise the cookies will be too hard.

$$b \geq s$$

- There should be at most 3 times more flour than sugar in the dough.

$$f \leq 3s$$

What are suitable amounts of sugar, butter and flour? Can the calories be further reduced?

Exercise 3: Arrays 4 Points

As we all know, Santa gives presents to children based on their behaviour over the last year. In fact, he keeps track of their good and bad deeds through several arrays where each array corresponds to a child and the positions correspond to good/bad ratings in certain categories. (Categories include cleaning up one's room, setting up the table, etc.)
In order to get all her wishes, Annika decided to be better than her brother Benno (the moral reference frame of a 10 year-old is limited).
Last year Annika managed to reach that goal:

$$\forall k. nicer(a[k], b[k])$$

However this year there were two updates in Santa's book (arrays):
In category i (which we won't mention here explicitly) Annika's goodness value has changed to v. In category j Benno's goodness value changed to w. It holds that $nicer(v, b[i])$ and $nicer(a[j], w)$.
Now the question that Annika asks herself is:

$$\begin{aligned} &(\forall k.\ nicer(a[k], b[k])) \wedge nicer(v, b[i]) \wedge nicer(a[j], w) \\ \rightarrow\ &(\forall k.\ nicer(a\langle i \lhd v\rangle[k], b\langle j \lhd w\rangle[k])) \end{aligned}$$

Is this formula *valid*?