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Hand in until December 4th, 2019
15:59 via the post boxes
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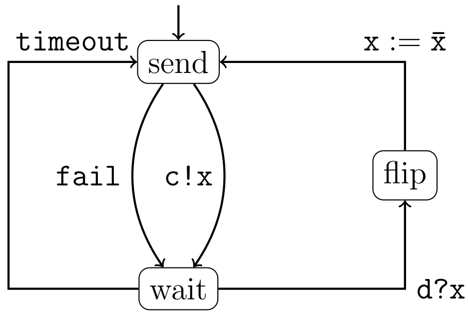
Tutorial for Cyber-Physical Systems - Discrete Models
Exercise Sheet 7

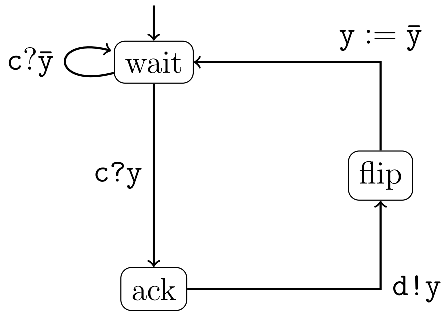
The goal of this exercise sheet is to get a better understanding of Cyber-Physical Systems represented by program graphs and parallel compositions of program graphs.

Exercise 1: Alternating Bit Protocol 12 Points

Consider the following variant of the Alternating Bit Protocol. Here, we omit the timer. The channel c is unreliable, while the channel d is reliable. Both channels have a limited capacity of just one message.



$\mathcal{P}_{Sender}$



$\mathcal{P}_{Receiver}$

The shared variables of $\mathcal{P}_{Sender}$ and $\mathcal{P}_{Receiver}$ are channels c and d. A channel may either be empty (which we denote, e.g., as $c = \varepsilon$) or contain a single message ($c = 0$ resp. $c = 1$). Additionally, the program graph $\mathcal{P}_{Sender}$ has the variable x and $\mathcal{P}_{Receiver}$ has the variable y. x and y are bits, i.e., they can have the values 0 or 1.

The actions behave as follows:

- The action `c!x` can only execute if the channel is empty ($c = \varepsilon$). If this is the case, then the channel contains the current value of x after the action is executed. For instance, if $x = 0$ before, then $c = 0$ (and still $x = 0$) afterwards.

  Analogously for `d!y`.

- The action `c?y` can only execute if the channel contains a message that is equal to the current value of y. If this is the case, the channel is empty ($c = \varepsilon$) afterwards. For instance, if $y = 1$ and $c = 1$, the action can execute and $c = \varepsilon$ afterwards. If $y = 0$ and $c = 1$, it cannot execute.

  Analogously for `d?x`.

- $\bar{x}$ denotes the negated value of x, e.g., if $x = 0$, then $\bar{x}$ is 1. The actions $\mathtt{x} := \bar{\mathtt{x}}$ and $\mathtt{y} := \bar{\mathtt{y}}$ behave as expected. The action $\mathtt{c?}\bar{\mathtt{y}}$ can only execute if the channel contains the value $\bar{y}$. If this is the case, the channel is empty ($c = \varepsilon$) afterwards. For instance, if $y = 1$ and $c = 0$, the action can execute and $c = \varepsilon$ afterwards.

- The actions `timeout` and `fail` can execute regardless of the values of the channel variables, and they do not modify the channel variables either.

Draw the following program graphs and transition systems. For the transition systems, assume that all channels are initially empty and the initial value of x and y is 0.

(a) Draw the (pure) interleaving $\mathcal{P}_{Sender} \mid\mid\mid \mathcal{P}_{Receiver}$ of the two program graphs.

(b) Draw the (reachable part) of the transition system $\mathcal{T}_{\mathcal{P}_{Sender} \mid\mid\mid \mathcal{P}_{Receiver}}$ for the program graph $\mathcal{P}_{Sender} \mid\mid\mid \mathcal{P}_{Receiver}$.

(c) Draw the reachable part of the transition system $\mathcal{T}_{\mathcal{P}_{Sender}}$.

(d) Draw the reachable part of the transition system $\mathcal{T}_{\mathcal{P}_{Receiver}}$.

(e) Draw the transition system $\mathcal{T}_{\mathcal{P}_{Sender}} \mid\mid\mid \mathcal{T}_{\mathcal{P}_{Receiver}}$. Is it the same as $\mathcal{T}_{\mathcal{P}_{Sender} \mid\mid\mid \mathcal{P}_{Receiver}}$?

Hint: The states of the transition systems in (b), (c) and (d) contain values for both variables c and d. The states in (b) additionally contain values for both x and y, while those in (c) and (d) only contain values for one of x and y. When we construct a transition system from a a program graph, we assume that the value of a variable can only change when an action is executed.

Exercise 2***:** 3 Bonus Points

As discussed in the lecture, the transition systems $\mathcal{T}_{\mathcal{P}_1} \mid\mid\mid \mathcal{T}_{\mathcal{P}_2}$ and $\mathcal{T}_{\mathcal{P}_1 \mid\mid\mid \mathcal{P}_2}$ for program graphs $\mathcal{P}_1$ and $\mathcal{P}_2$ may in general be different (and they often are). However, there exist program graphs $\mathcal{P}_1$ and $\mathcal{P}_2$ where the two transition systems are equal. Give an example of two such program graphs.