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Hand in until October 30th, 2019
15:59 via the post boxes
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Tutorial for Cyber-Physical Systems - Discrete Models
Exercise Sheet 2

The goal of this sheet to prepare notions that we will use in the context of transition systems. We will not introduce all of these notions explicitly in the lecture. We assume that you either have seen them before (e.g., in other lectures) or are able to find the necessary definitions in the literature yourself. In particular, we recommend the book Principles of Model Checking *by Christel Baier and Joost-Pieter Katoen (in future referred to as* "the book"*), on which the lecture is based. A copy is available in the library*[1]*. The purpose of these exercises is to give you the opportunity to practice working with these definitions and get feedback on your results.*

Exercise 1: Set Notation for Valuations 6 Points

This exercise refers to Sect. A.3 Propositional Logic in the appendix of the book. The goal of this exercise is to train your understanding of valuations and when a valuation satisfies a Boolean formula (and when it does not). Given a valuation μ and a Boolean formula ϕ, it is easy to see whether the valuation satisfies the Boolean formula, formally $\mu \models \phi$. It is perhaps more difficult to think about all valuations that satisfy a given Boolean formula. However, this will become relevant when we deal with transition systems, where the states will be labeled with valuations.

Let $AP = \{a_1, \dots, a_n\}$ be a set of atomic propositions (if you prefer, you can say Boolean variable instead of atomic proposition).
Intuitively, a valuation gives a value to each atomic proposition. The value is a truth value, which we here denote by 0 or 1. The book uses the term "evaluation" instead of "valuation". Formally, a valuation can be represented as a function $\mu : AP \to \{0, 1\}$. If $\mu(a_i)$ is 0, then the truth value of the atomic proposition a_i is 0. Alternatively, a valuation can be represented by a subset A of atomic propositions. If $a_i \in A$, then the truth value of the atomic proposition a_i is 1 (and 0, otherwise). The connection between the two representations can be stated by $A_\mu = \{a \in AP \mid \mu(a) = 1\}$. See also the book.

(a) Consider the atomic propositions *"the battery is empty"* (a_1), *"the generator is on"* (a_2) and *"the tank is empty"* (a_3). List explicitly all valuations μ over $AP = \{a_1, a_2, a_3\}$ such that $\mu \models \phi_1$, once expressed as functions and once expressed as sets. Do the same for all valuations μ such that $\mu \models \phi_1 \wedge \phi_2$.

- $\phi_1 = a_1 \to a_2$ (*"If the battery is empty, the generator is on."*)
- $\phi_2 = a_3 \to \neg a_2$ (*"If the tank is empty, the generator is off."*)

[1] `https://katalog.ub.uni-freiburg.de/opac/RDSIndex/Search?type0[]=ta&lookfor0[]=Principles+of+Model+Checking`

(b) Give a description of all valuations μ such that $\mu \models \phi$, once expressed in terms of functions and once in terms of sets, with

- $\phi = a_1 \wedge \cdots \wedge a_i$
- $\phi = a_1 \vee \cdots \vee a_i$

where i is some number smaller than n, i.e. , $i \leq n$.

(c) Let $AP = \{a_1, a_2, a_3, a_4, a_5\}$. Find a formula ϕ such that μ_A with $A = \{a_2, a_3\}$ is the *unique* satisfying valuation for ϕ.

Exercise 2: Traffic Lights 4 Points

The goal of this exercise is to train reasoning about the behaviour of transition systems, and perhaps discover some of the pitfalls. The lecture will later introduce precise formalisms for questions like these, as well as algorithms that can answer them correctly.

In the lecture, we discussed a control system for a pedestrian traffic light. Answer the following questions, and give a short explanation for your answer.

- Is it always true, that when the pedestrian presses the request button (and the `request!` event occurs), the pedestrian light will be red (i.e., is in state `don't walk`) in the next step?
- Is there an infinite sequence of events in which, from some point on, the traffic light (for the cars) stays red forever?
- Can the system reach a configuration where both the pedestrian light and the traffic light are green, i.e., in state `walk` resp. state `green`, at the same time?
- Can the system reach a configuration where both the pedestrian light and the traffic light are red, i.e., in state `don't walk` resp. state `red`, at the same time?

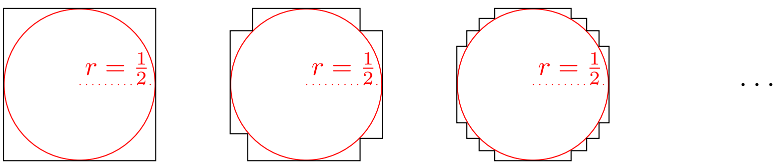
Exercise 3*: Correct Proofs 2 Bonus Points

The goal of this task is to initiate philosophical discussions about mathematics.

Consider the following proof sketch.

We use the fact that the perimeter of a circle of radius $r = 0.5$ is π to prove that π is equal to 4. Consider the sequence of polygons depicted below. It converges to the circle. Since each polygon has the perimeter $P_i = 4$ for $i = 1, 2, \ldots$ and the limit of a constant sequence is the corresponding constant, the perimeter of the circle must be 4. Hence, π is equal to 4.

Do you trust the proof?



$P_1 = 4$ $P_2 = 4$ $P_3 = 4$