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Hand in until December 16th, 2020
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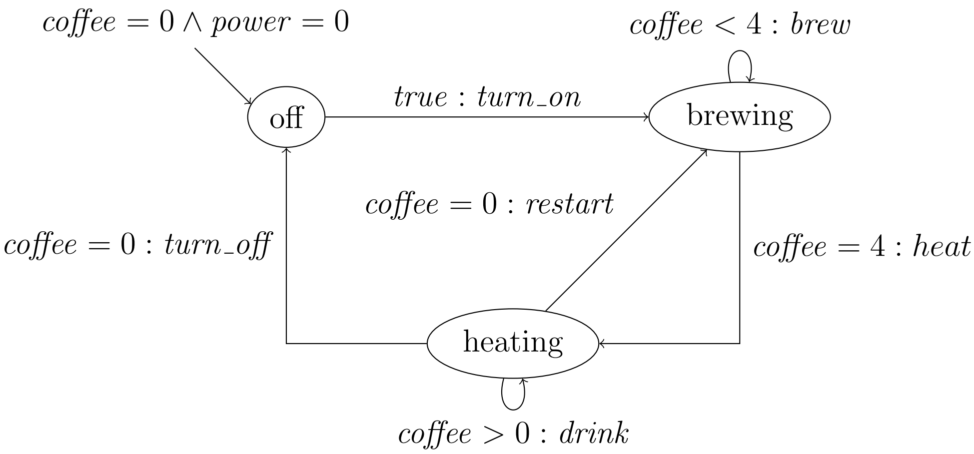
Tutorial for Cyber-Physical Systems - Discrete Models

Exercise Sheet 6

Exercise 1: Coffee Machine and Transition System — 8 Points

The goal of this task is to provide some intuition on when the system described by a program graph satisfies given properties, by looking at the transition system.

The following program graph describes a simple coffee machine:



The effect of the operations is given by:

$$
\begin{aligned}
\mathit{Effect}(turn\_on, \eta) &= \eta[power := 1] \\
\mathit{Effect}(turn\_off, \eta) &= \eta[power := 0] \\
\mathit{Effect}(brew, \eta) &= \eta[coffee := coffee + 1] \\
\mathit{Effect}(drink, \eta) &= \eta[coffee := coffee - 1] \\
\mathit{Effect}(restart, \eta) &= \eta \\
\mathit{Effect}(heat, \eta) &= \eta
\end{aligned}
$$

(a) Give the program from which the above program graph is derived. Use the Guarded Command Language (GCL) for the program. Mark the lines of the program that correspond to the locations off, brewing, and heating.

(b) Draw the (reachable part of the) transition system corresponding to the program graph. Choose 5 transitions of the transition system, and justify their existence using the respective SOS-rule.

Use the SOS-rules to argue why the following transitions are *not* part of the transition system:

$$\langle \mathsf{off}, \{\mathit{coffee} \mapsto 0, \mathit{power} \mapsto 0\}\rangle \xrightarrow{\mathit{heat}} \langle \mathsf{heating}, \{\mathit{coffee} \mapsto 0, \mathit{power} \mapsto 0\}\rangle$$
$$\langle \mathsf{brewing}, \{\mathit{coffee} \mapsto 4, \mathit{power} \mapsto 1\}\rangle \xrightarrow{\mathit{brew}} \langle \mathsf{brewing}, \{\mathit{coffee} \mapsto 5, \mathit{power} \mapsto 1\}\rangle$$

(c) Use the transition system to explain which of the following properties are true for every execution of the coffee machine.

(i) If the machine is turned off ($\mathit{power} = 0$), it contains no coffee ($\mathit{coffee} = 0$).

(ii) If there are two cups of coffee ($\mathit{coffee} = 2$), there are either three or four cups of coffee in the next step ($\mathit{coffee} = 3$, $\mathit{coffee} = 4$).

(iii) There are always at most four cups of coffee ($\mathit{coffee} \leq 4$).

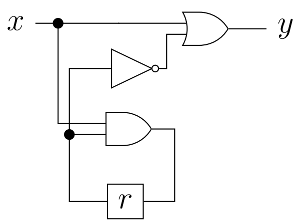
(iv) The coffee machine will be turned off (i.e., in location *off*) infinitely often.

(v) If there is no coffee ($\mathit{coffee} = 0$), there will be coffee after at most three steps.

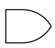
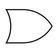
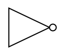
Exercise 2: Hardware Circuit and Transition System 4 Points

The goal of this exercise is to go from a pictorial representation of a hardware system to a formal model.

Consider the following sequential hardware circuit.



Draw the transition system of the hardware circuit. That is, the states are the valuations of the input x and the register r. The transitions represent the stepwise behavior where the value of the input bit x may or may not change in each step.
You may assume that initially the register r has the value `false`.

For your reference:  = AND gate,  = OR gate,  = NOT gate