Tutorial for Cyber-Physical Systems - Discrete Models
Exercise Sheet 7

Please remember to register for both: the exam (deadline: January 17th) and the "Studienleistung" (deadline: February 13th) in HisInOne.

Exercise 1: Starvation Freedom
In the lecture we discussed two different definitions of the starvation freedom property for the mutual exclusion problem. We consider the set of atomic propositions $AP = \{wait_1, wait_2, crit_1, crit_2\}$. The properties are defined as

$$LIVE := \left\{ \begin{array}{l}
(\exists^\infty i \in \mathbb{N}. \text{wait}_1 \in A_i) \rightarrow (\exists^\infty i \in \mathbb{N}. \text{crit}_1 \in A_i) \\
(\exists^\infty i \in \mathbb{N}. \text{wait}_2 \in A_i) \rightarrow (\exists^\infty i \in \mathbb{N}. \text{crit}_2 \in A_i)
\end{array} \right. \text{ s.t.}$$

$$LIVE' := \left\{ \begin{array}{l}
\forall i \in \mathbb{N}.(\text{wait}_1 \in A_i \rightarrow (\exists j \in \mathbb{N}. j \geq i \land \text{crit}_1 \in A_j)) \\
\forall i \in \mathbb{N}.(\text{wait}_2 \in A_i \rightarrow (\exists j \in \mathbb{N}. j \geq i \land \text{crit}_2 \in A_j))
\end{array} \right. \text{ s.t.}$$

where $\exists^\infty$ means “there exist infinitely many”.

(a) Show that the property $LIVE'$ is at least as strong as the property $LIVE$, i.e., prove that $LIVE' \subseteq LIVE$.

(b) Show that $LIVE'$ is a strictly stronger property than $LIVE$: Give an infinite trace $\pi = A_0A_1A_2\ldots$, and prove that $\pi \in LIVE$ but $\pi \notin LIVE'$.

(c) Does such a trace $\pi$ with $\pi \in LIVE$ but $\pi \notin LIVE'$ exist in the transition systems for mutual exclusion discussed in the lecture (with semaphore resp. with Peterson algorithm)? Why/why not?

(d) Does there exist a trace $\pi$ with $\pi \in LIVE'$ but $\pi \notin LIVE$ in the transition systems for mutual exclusion discussed in the lecture (with semaphore resp. with Peterson algorithm)? Why/why not?
Exercise 2: Trace Inclusion
Consider the program graphs $P_1, P_2, P_{3a},$ and $P_{3b}$ as well as the transition system $T_4$.

The domain of the variable $x$ in all 3 program graphs is the set of integers $\mathbb{Z}$. The effect of the assignment action is as expected, and $\text{Effect}(\text{nop}, \eta) = \eta$.

(a) Draw the (reachable part of the) transition systems $T_{P_1}, T_{P_2}$ and $T_{P_{3a} \parallel P_{3b}}$.

As atomic propositions of the transition system, use the guards of the actions in the program graph, i.e. $AP = \{x > 0, x = 0\}$.

(b) For each pair $T, T' \in \{T_{P_1}, T_{P_2} T_{P_{3a} \parallel P_{3b}}, T_4\}$, consider the trace inclusion $\text{Traces}(T) \subseteq \text{Traces}(T')$. If it holds, argue why this is the case. If it does not hold, give a trace $\pi = A_0A_1A_2 \ldots$ such that $\pi \in \text{Traces}(T)$ but $\pi \notin \text{Traces}(T')$.

(c) Give a property $E$ (i.e., a set of traces) such that $T_{P_1} \models E$ and $T_{P_2} \models E$ but $T_{P_{3a} \parallel P_{3b}} \not\models E$ and $T_4 \not\models E$. Give traces of $T_{P_{3a} \parallel P_{3b}}$ and $T_4$ that violate the property, and argue why $T_{P_1}$ and $T_{P_2}$ satisfy the property.

Reminder: A transition system $T$ satisfies a property $E$ (i.e. $T \models E$) if and only if $\text{Traces}(T) \subseteq E$.