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## Tutorial for Cyber-Physical Systems - Discrete Models Exercise Sheet 7

Please remember to register for both: the exam (deadline: January 17th) and the "Studienleistung" (deadline: February 13th) in HisInOne.

## **Exercise 1: Starvation Freedom**

4 Points

In the lecture we discussed two different definitions of the starvation freedom property for the mutual exclusion problem. We consider the set of atomic propositions  $AP = \{wait_1, wait_2, crit_1, crit_2\}$ . The properties are defined as

 $LIVE := \begin{cases} \text{set of all infinite traces } A_0A_1A_2\dots \text{ s.t.} \\ (\exists^{\infty}i \in \mathbb{N} . \mathsf{wait}_1 \in A_i) \to \exists^{\infty}i \in \mathbb{N} . \mathsf{crit}_1 \in A_i \\ (\exists^{\infty}i \in \mathbb{N} . \mathsf{wait}_2 \in A_i) \to \exists^{\infty}i \in \mathbb{N} . \mathsf{crit}_2 \in A_i \end{cases}$  $LIVE' := \begin{cases} \text{set of all infinite traces } A_0A_1A_2\dots \text{ s.t.} \\ \forall i \in \mathbb{N} . (\mathsf{wait}_1 \in A_i \to \exists j \in \mathbb{N} . j \ge i \land \mathsf{crit}_1 \in A_j) \\ \forall i \in \mathbb{N} . (\mathsf{wait}_2 \in A_i \to \exists j \in \mathbb{N} . j \ge i \land \mathsf{crit}_2 \in A_j) \end{cases}$ 

where  $\exists^{\infty}$  means "there exist infinitely many".

- (a) Show that the property LIVE' is at least as strong as the property LIVE, i.e., prove that  $LIVE' \subseteq LIVE$ .
- (b) Show that LIVE' is a strictly stronger property than LIVE: Give an infinite trace  $\pi = A_0A_1A_2...$ , and prove that  $\pi \in LIVE$  but  $\pi \notin LIVE'$ .
- (c) Does such a trace  $\pi$  with  $\pi \in LIVE$  but  $\pi \notin LIVE'$  exist in the transition systems for mutual exclusion discussed in the lecture (with semaphore resp. with Peterson algorithm)? Why/why not?
- (d) Does there exist a trace  $\pi$  with  $\pi \in LIVE'$  but  $\pi \notin LIVE$  in the transition systems for mutual exclusion discussed in the lecture (with semaphore resp. with Peterson algorithm)? Why/why not?

## Exercise 2: Trace Inclusion

9 Points

Consider the program graphs  $\mathcal{P}_1$ ,  $\mathcal{P}_2$ ,  $\mathcal{P}_{3a}$ , and  $\mathcal{P}_{3b}$  as well as the transition system  $\mathcal{T}_4$ .



The domain of the variable x in all 3 program graphs is the set of integers  $\mathbb{Z}$ . The effect of the assignment action is as expected, and  $Effect(nop, \eta) = \eta$ .

(a) Draw the (reachable part of the) transition systems  $\mathcal{T}_{\mathcal{P}_1}$ ,  $\mathcal{T}_{\mathcal{P}_2}$  and  $\mathcal{T}_{\mathcal{P}_{3a} \parallel \mid \mathcal{P}_{3b}}$ . As atomic propositions of the transition system, use the guards of the actions in

As atomic propositions of the transition system, use the guards of the act the program graph, i.e.  $AP = \{x > 0, x = 0\}.$ 

- (b) For each pair  $\mathcal{T}, \mathcal{T}' \in \{\mathcal{T}_{\mathcal{P}_1}, \mathcal{T}_{\mathcal{P}_2}\mathcal{T}_{\mathcal{P}_{3a}|||\mathcal{P}_{3b}}, \mathcal{T}_4\}$ , consider the trace inclusion  $Traces(\mathcal{T}) \subseteq Traces(\mathcal{T}')$ . If it holds, argue why this is the case. If it does not hold, give a trace  $\pi = A_0 A_1 A_2 \dots$  such that  $\pi \in Traces(\mathcal{T})$  but  $\pi \notin Traces(\mathcal{T}')$ .
- (c) Give a property E (i.e., a set of traces) such that  $\mathcal{T}_{\mathcal{P}_1} \models E$  and  $\mathcal{T}_{\mathcal{P}_2} \models E$  but  $\mathcal{T}_{\mathcal{P}_{3a} \parallel \mathcal{P}_{3b}} \not\models E$  and  $\mathcal{T}_4 \not\models E$ . Give traces of  $\mathcal{T}_{\mathcal{P}_{3a} \parallel \mathcal{P}_{3b}}$  and  $\mathcal{T}_4$  that violate the property, and argue why  $\mathcal{T}_{\mathcal{P}_1}$  and  $\mathcal{T}_{\mathcal{P}_2}$  satisfy the property. **Reminder**: A transition system  $\mathcal{T}$  satifies a property E (i.e.  $\mathcal{T} \models E$ ) if and only if  $Traces(\mathcal{T}) \subseteq E$ .