Exercise 1: Prefixes and Closure of a Property

The goal of this task is to get a better understanding of the relation between the set of finite prefixes of a property and the closure (which is defined using the prefixes).

Let $P$ be an LT property. Prove the following claims:

(a) $P \subseteq \text{cl}(P)$

(b) $\text{pref}(\text{cl}(P)) = \text{pref}(P)$ (Hint: You may use the property from a) here)

Exercise 2: Safety Properties

The goal of this task is to learn how to recognize safety properties and to understand prefixes and closures.

Consider following properties over the set $AP = \{a, b\}$ of atomic propositions.

- $P_1 = \{A_0A_1A_2\ldots \mid \forall i. a \not\in A_i\}$
  (a should never occur)
- $P_2 = \{A_0A_1A_2\ldots \mid \exists i. (a \in A_i \land \forall j \neq i. a \not\in A_j)\}$
  (a should occur exactly once)
- $P_3 = \{A_0A_1A_2\ldots \mid \forall i. (A_{2i} = \{a\} \land A_{2i+1} = \{b\})\}$
  (a and b alternate, starting with a)
- $P_4 = \{A_0A_1A_2\ldots \mid \forall i. (a \in A_i \rightarrow \exists j \geq i. b \in A_j)\}$
  (every a should eventually be followed by b)
- $P_5 = \{A_0A_1A_2\ldots \mid \forall i. (b \in A_i \rightarrow a \in A_i)\}$
  (every time b holds, a also holds)
- $P_6 = \{A_0A_1A_2\ldots \mid \forall i. (b \in A_i \rightarrow \forall j \neq i. b \not\in A_j)\}$
  (b holds at most once)

For each property $P_i$ complete the following tasks:

(a) Give the set of prefixes, i.e. $\text{pref}(P_i)$.

(b) Give the set of bad prefixes, i.e. $\text{BadPref}_{P_i}$

(c) Provide its closure, i.e. $\text{cl}(P_i)$.

(d) Determine if $P_i$ is an invariant. In that case provide the invariant condition.

(e) Determine if $P_i$ is a safety property and explain why.