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Hand in until February 3rd, 2021
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Tutorial for Cyber-Physical Systems - Discrete Models
Exercise Sheet 11

Exercise 1: Prefixes and Closure of a Property   3 Points

The goal of this task is to get a better understanding of the relation between the set of finite prefixes of a property and the closure (which is defined using the prefixes).
Let P be an LT property. Prove the following claims:

(a) $P \subseteq cl(P)$

(b) $pref(cl(P)) = pref(P)$ (Hint: You may use the property from a) here)

Exercise 2: Safety Properties   15 Points

The goal of this task is to learn how to recognize safety properties and to understand prefixes and closures.
Consider following properties over the set $AP = \{a, b\}$ of atomic propositions.

- $P_1 = \{A_0 A_1 A_2 \ldots \mid \forall i.\ a \notin A_i\}$
  (a should never occur)

- $P_2 = \{A_0 A_1 A_2 \ldots \mid \exists i.\ (a \in A_i \land \forall j \neq i.\ a \notin A_j)\}$
  (a should occur exactly once)

- $P_3 = \{A_0 A_1 A_2 \ldots \mid \forall i.\ (A_{2i} = \{a\} \land A_{2i+1} = \{b\})\}$
  (a and b alternate, starting with a)

- $P_4 = \{A_0 A_1 A_2 \ldots \mid \forall i.\ (a \in A_i \to \exists j \geq i.\ b \in A_j)\}$
  (every a should eventually be followed by b)

- $P_5 = \{A_0 A_1 A_2 \ldots \mid \forall i.\ (b \in A_i \to a \in A_i)\}$
  (every time b holds, a also holds)

- $P_6 = \{A_0 A_1 A_2 \ldots \mid \forall i.\ (b \in A_i \to \forall j \neq i.\ b \notin A_j)\}$
  (b holds at most once)

For each property P_i complete the following tasks:

(a) Give the set of prefixes, i.e. $pref(P_i)$.

(b) Give the set of bad prefixes, i.e. $BadPref_{P_i}$

(c) Provide its closure, i.e. $cl(P_i)$.

(d) Determine if P_i is an invariant. In that case provide the invariant condition.

(e) Determine if P_i is a safety property and explain why.