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Hand in until November 10th, 2021
23:59 via ILIAS
Discussion: November 15th / 16th, 2021

Tutorial for Cyber-Physical Systems - Discrete Models
Exercise Sheet 3

Exercise 1: Intersection of ω-regular languages — 6 Points

The goal of this exercise is to create Büchi automata that accept the intersection of two ω-regular languages, as we are interested in the intersections in the lecture later on.

In the following exercise we are interested in the intersection of two ω-regular languages. The intersection is also an ω-regular language, so we can represent it in a Büchi automaton. We are asking you to first look at the language obtained by taking the intersection of the two given languages, and then construct the Büchi automaton for that language.

There is also an algorithm to intersect two Büchi automata. In fact, the exercise will help you to gain an intuition about the algorithm to intersect two Büchi automata. If you wish, you can try to figure out the algorithm on your own. **Be aware, however. One cannot simply take the construction of the finite automaton for the intersection you have seen in the exercise session (what will go wrong?).**

Given two ω-languages L_1 and L_2, a Büchi automaton A accepts the intersection of L_1 and L_2 if and only if $L_\omega(A) = L_1 \cap L_2$. This means a word $w \in \Sigma^\omega$ is accepted by A if and only if $w \in L_1$ and $w \in L_2$.

(a) Consider the following ω-languages over the alphabet $\Sigma = \{a, b, c\}$.

$$\begin{aligned} L_1 &= \{\, x_0x_1\ldots \mid \overset{\infty}{\forall} i \in \mathbb{N}_0 \,.\, x_i \neq a\} \\ &= \{\, x_0x_1\ldots \mid \exists j \in \mathbb{N}_0 \,.\, \forall i \in \mathbb{N}_0 \,.\, i > j \rightarrow x_i \neq a\} \\ L_2 &= \{\, x_0x_1\ldots \mid \overset{\infty}{\exists} i \in \mathbb{N}_0 \,.\, x_i = b\,\} \\ &= \{\, x_0x_1\ldots \mid \forall j \in \mathbb{N}_0 \,.\, \exists i \in \mathbb{N}_0 \,.\, i > j \wedge x_i = b\,\} \end{aligned}$$

Draw a Büchi automaton A that accepts the intersection of L_1 and L_2.

(b) Consider the following ω-languages over the alphabet $\Sigma = \{a, b\}$.

$$\begin{aligned} L_1 &= \{\, x_0x_1\ldots \mid \overset{\infty}{\exists} i \in \mathbb{N}_0 \,.\, x_i = a\,\} \\ &= \{\, x_0x_1\ldots \mid \forall j \in \mathbb{N}_0 \,.\, \exists i \in \mathbb{N}_0 \,.\, i > j \wedge x_i = a\,\} \\ L_2 &= \{\, x_0x_1\ldots \mid \overset{\infty}{\exists} i \in \mathbb{N}_0 \,.\, x_i = b\,\} \\ &= \{\, x_0x_1\ldots \mid \forall j \in \mathbb{N}_0 \,.\, \exists i \in \mathbb{N}_0 \,.\, i > j \wedge x_i = b\,\} \end{aligned}$$

Draw a Büchi automaton A that accepts the intersection of L_1 and L_2.

(c) Consider the following ω-languages over the alphabet $\Sigma = \{a, b\}$.

$$\begin{aligned} L_1 &= \{\, x_0 x_1 \ldots \mid \overset{\infty}{\forall} i \in \mathbb{N}_0 \,.\, x_i \neq a \} \\ &= \{\, x_0 x_1 \ldots \mid \exists j \in \mathbb{N}_0 \,.\, \forall i \in \mathbb{N}_0 \,.\, i > j \to x_i \neq a \} \\ L_2 &= \{\, x_0 x_1 \ldots \mid \forall i \in \mathbb{N}_0 \,.\, x_i = b \to x_{i+1} = a \,\} \end{aligned}$$

Draw a Büchi automaton A that accepts the intersection of L_1 and L_2.

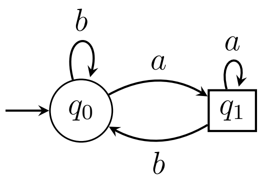
Exercise 2: Intersection of Büchi automata 6 Points

The goal of this exercise is to construct a Büchi automaton for the intersection Büchi automata.

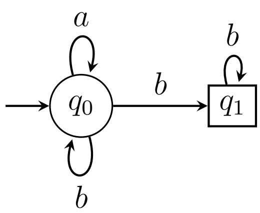
In this exercise you should think about the accepted language of the intersection of given Büchi automata. And again: It should be easier to create an automaton for the intersected languages than applying the algorithm.

Be careful: The construction of the Büchi automaton for the intersection is different from the construction of the finite automaton for the intersection you have seen in the exercise session!

Consider the following Büchi automaton A.

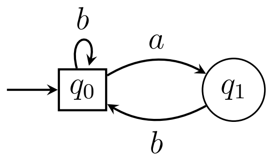


(a) Consider the following Büchi automaton A_1



Describe the language $L_\omega(A) \cap L_\omega(A_1)$ (i.e. all words that are accepted by A and A_1) in set notation and draw a Büchi automaton that accepts this language.

(b) Consider the following Büchi automaton A_2



Describe the language $L_\omega(A) \cap L_\omega(A_2)$ (i.e. all words that are accepted by A and A_1) in set notation and draw a Büchi automaton that accepts this language.

Exercise 3: Transition Systems 6 Points

The goal of this exercise is to understand the connection between mathematical notation and graphical representation of transition systems.

In the lecture, we went over the labeling of edges and the labeling of nodes rather quickly, so here is a refresher:

- If a transition from s to s' is labeled by the action α, then we have the triple (s, α, s') in the transition relation $\longrightarrow$.
- If the state s is labeled by the set of atomic propositions $\{A_1, \ldots, A_n\}$, then the image of s under the labeling function L is $\{A_1, \ldots, A_n\}$, formally $L(s) = \{A_1, \ldots, A_n\}$.
- Note that the set of atomic propositions $\{A_1, \ldots, A_n\}$ can be empty (which corresponds to $n = 0$).

(a) Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a transition system with

- the set of states $S = \{\texttt{locked}, \texttt{checking}, \texttt{opened}\}$,
- the set of action $Act = \{\texttt{insert\_ticket}, \texttt{unlock}, \texttt{enter}, \texttt{error}\}$,
- the transition relation

$$\rightarrow = \{(\texttt{locked}, \texttt{insert\_ticket}, \texttt{checking}), (\texttt{checking}, \texttt{unlock}, \texttt{opened}), \\ (\texttt{opened}, \texttt{enter}, \texttt{locked}), (\texttt{checking}, \texttt{error}, \texttt{locked})\},$$

- the initial states $S_0 = \{\texttt{locked}\}$,
- the set of atomic propositions $AP = \{\texttt{light\_red}, \texttt{light\_green}\}$,
- and the labeling function L with $L(\texttt{locked}) = \{\texttt{light\_red}\}$, $L(\texttt{checking}) = \emptyset$ and $L(\texttt{opened}) = \{\texttt{light\_green}\}$.

Draw this transition system. Can you see what cyber-physical system it models?

(b) The transition system shown below models an elevator. Give the corresponding mathematical definition, i.e., define the tuple $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ that is described by the picture, in the style of (a).
In which states is the elevator door closed?