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Hand in until December 1st, 2021
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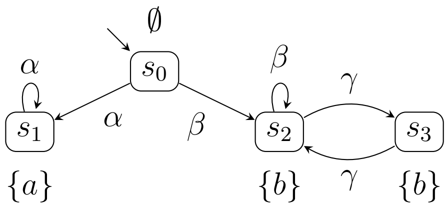
Discussion: December 6th/7th, 2021

Tutorial for Cyber-Physical Systems - Discrete Models
Exercise Sheet 6

Exercise 1: Executions, Paths and Traces 5+2 Points

Consider the following transition system with the set of atomic propositions $AP = \{a, b\}$.



Solve the following tasks.

(a) Give examples that illustrate the difference between the different notions of executions and execution fragments. Therefore give the following execution fragments of the given transition system:

- An execution fragment that is neither initial nor maximal
- An initial execution fragment that is not maximal
- A maximal execution fragment that is not initial
- An initial and maximal execution fragment (i.e. an execution)

(b) How many executions does the transition system have? Explain your answer.

(c) Provide a path of the transition system. How many are there in total?

(d) How many traces does the transition system have? Provide all of them.

(e) **Bonus:** Is it possible to have a transition system with infinitely many executions and only finitely many paths? If yes, provide such a transition system, otherwise explain why this is not possible.

Exercise 2: Starvation Freedom 4 Points

In the lecture we discussed two different definitions of the starvation freedom property for the mutual exclusion problem. We consider the set of atomic propositions $AP = \{\mathsf{wait}_1, \mathsf{wait}_2, \mathsf{crit}_1, \mathsf{crit}_2\}$. The properties are defined as

$$LIVE := \begin{cases} \text{set of all infinite traces } A_0A_1A_2\ldots \text{ s.t.} \\ (\overset{\infty}{\exists} i \in \mathbb{N} \,.\, \mathsf{wait}_1 \in A_i) \to \overset{\infty}{\exists} i \in \mathbb{N} \,.\, \mathsf{crit}_1 \in A_i \\ (\overset{\infty}{\exists} i \in \mathbb{N} \,.\, \mathsf{wait}_2 \in A_i) \to \overset{\infty}{\exists} i \in \mathbb{N} \,.\, \mathsf{crit}_2 \in A_i \end{cases}$$

$$LIVE' := \begin{cases} \text{set of all infinite traces } A_0A_1A_2\ldots \text{ s.t.} \\ \forall i \in \mathbb{N} \,.\, (\mathsf{wait}_1 \in A_i \to \exists j \in \mathbb{N} \,.\, j \geq i \wedge \mathsf{crit}_1 \in A_j) \\ \forall i \in \mathbb{N} \,.\, (\mathsf{wait}_2 \in A_i \to \exists j \in \mathbb{N} \,.\, j \geq i \wedge \mathsf{crit}_2 \in A_j) \end{cases}$$

(a) Show that the property $LIVE'$ is at least as strong as the property $LIVE$, i.e., prove that $LIVE' \subseteq LIVE$.

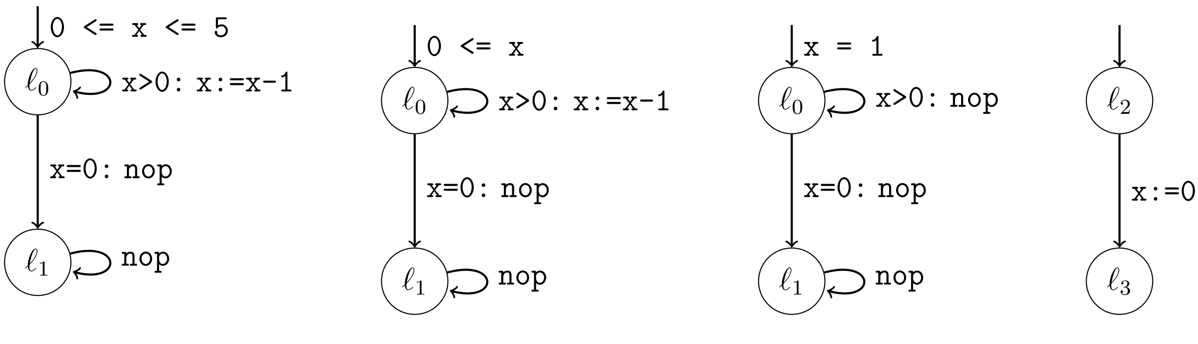
(b) Show that $LIVE'$ is a *strictly stronger* property than $LIVE$: Give an infinite trace $\pi = A_0A_1A_2\ldots$, and prove that $\pi \in LIVE$ but $\pi \notin LIVE'$.

(c) Does such a trace π with $\pi \in LIVE$ but $\pi \notin LIVE'$ exist in the transition systems for mutual exclusion discussed in the lecture (with semaphore resp. with Peterson algorithm)? Why/why not?

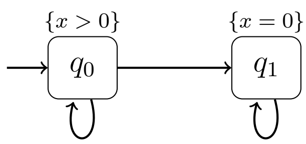
(d) Does there exist a trace π with $\pi \in LIVE'$ but $\pi \notin LIVE$ in the transition systems for mutual exclusion discussed in the lecture (with semaphore resp. with Peterson algorithm)? Why/why not?

Exercise 3: Trace Inclusion 9 Points

Consider the program graphs $\mathcal{P}_1$, $\mathcal{P}_2$, $\mathcal{P}_{3a}$, and $\mathcal{P}_{3b}$ as well as the transition system $\mathcal{T}_4$.



$\mathcal{P}_1$ $\mathcal{P}_2$ $\mathcal{P}_{3a}$ $\mathcal{P}_{3b}$



$\mathcal{T}_4$

The domain of the variable x in all 3 program graphs is the set of integers $\mathbb{Z}$. The effect of the assignment action is as expected, and $\mathit{Effect}(\texttt{nop}, \eta) = \eta$.

(a) Draw the (reachable part of the) transition systems $\mathcal{T}_{\mathcal{P}_1}$, $\mathcal{T}_{\mathcal{P}_2}$ and $\mathcal{T}_{\mathcal{P}_{3a}|||\mathcal{P}_{3b}}$.

As atomic propositions of the transition system, use the guards of the actions in the program graph, i.e. $AP = \{x > 0, x = 0\}$.

(b) For each pair $\mathcal{T}, \mathcal{T}' \in \{\mathcal{T}_{\mathcal{P}_1}, \mathcal{T}_{\mathcal{P}_2}\mathcal{T}_{\mathcal{P}_{3a}|||\mathcal{P}_{3b}}, \mathcal{T}_4\}$, consider the trace inclusion $\mathit{Traces}(\mathcal{T}) \subseteq \mathit{Traces}(\mathcal{T}')$. If it holds, argue why this is the case. If it does not hold, give a trace $\pi = A_0A_1A_2\ldots$ such that $\pi \in \mathit{Traces}(\mathcal{T})$ but $\pi \notin \mathit{Traces}(\mathcal{T}')$.

(c) Give a property E (i.e., a set of traces) such that $\mathcal{T}_{\mathcal{P}_1} \models E$ and $\mathcal{T}_{\mathcal{P}_2} \models E$ but $\mathcal{T}_{\mathcal{P}_{3a}|||\mathcal{P}_{3b}} \not\models E$ and $\mathcal{T}_4 \not\models E$. Give traces of $\mathcal{T}_{\mathcal{P}_{3a}|||\mathcal{P}_{3b}}$ and $\mathcal{T}_4$ that violate the property, and argue why $\mathcal{T}_{\mathcal{P}_1}$ and $\mathcal{T}_{\mathcal{P}_2}$ satisfy the property.
Reminder: A transition system $\mathcal{T}$ satifies a property E (i.e. $\mathcal{T} \models E$) if and only if $\mathit{Traces}(\mathcal{T}) \subseteq E$.