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Tutorial for Cyber-Physical Systems - Discrete Models
Exercise Sheet 13

Exercise 1: Equivalence of LTL formulas 8 Points + 2 Bonus Points

Consider the following claims about equivalences of LTL formulas.
Provide a counterexample (i.e. a transition system that satisfies one of the properties and violates the other) if an equivalence does not hold.

(a) $\Box a \wedge \bigcirc \Diamond a \overset{?}{\equiv} \Box a$

(b) $\Diamond a \wedge \bigcirc \Box a \overset{?}{\equiv} \Diamond a$

(c) $\Box a \rightarrow \Diamond b \overset{?}{\equiv} a \ \mathsf{U} \ (b \vee \neg a)$

(d) $a \ \mathsf{U} \ \mathit{false} \overset{?}{\equiv} \Box a$

(e) $\Box \bigcirc b \overset{?}{\equiv} \Box b$

Bonus: If an equivalence holds, give a proof.

Exercise 2: Positive Normal Form 4 Points

Transform the following LTL formulas into an equivalent LTL formulas in PNF.

(a) $\neg\Big((\Box a) \rightarrow \big((a \wedge \neg c) \ \mathsf{U} \ \neg(\bigcirc b)\big)\Big)$

(b) $\neg\Big(\Diamond a \wedge \bigcirc(\neg(\Box b \rightarrow c))\Big)$

Exercise 3*: LTL Equivalence 3 Bonus Points

Let φ, ψ be two LTL formulas. Prove the following statement from the lecture:

$$\mathit{Words}(\varphi) = \mathit{Words}(\psi)$$
iff
for all transition systems $\mathcal{T}$: $\mathcal{T} \models \varphi \iff \mathcal{T} \models \psi$

Hint: A transition system $\mathcal{T}$ may have infinitely many states.