Prof. Dr. Andreas Podelski
Dominik Klumpp
Elisabeth Henkel

Hand in until October 26th, 2022
23:59 via ILIAS
Discussion: Oct 31st / Nov 1st, 2022

Tutorial for Cyber-Physical Systems - Discrete Models

Exercise Sheet 1

General comments on our exercises

- First, try to understand the problem on your own. Then discuss the problem (resp. your solution) with your fellow students. Finally, write down the solution alone or in groups of two.
- The exercises are **not optional**. You must obtain at least 50% of the exercise points. The goal is to train you to write down things in a formally correct way (being able to write down things in a formally correct way is an important goal of this lecture, and you need to train it). Correcting the exercises will help us to evaluate your knowledge and to evaluate your capability to solve the exercises in the exam.
- The mathematical background of this course's participants is very heterogeneous. Don't get frustrated if fellow students solve exercises quicker than you do.

Exercise 1: Propositional Logic — 6 Points

The goal of this exercise is to train how to formulate statements as logical formulas. We assume a background knowledge in propositional logic. If you are not familiar with it, or you need a refresher, take a look at appendix A.3 of the book Principles of Model Checking *by Christel Baier and Joost-Pieter Katoen on which the lecture is based (a copy is available in the library[1]), or one of many other resources available online.*

Alice, Bob and Claire want to attend the CPS I lecture. The exercise groups are almost full, only group 1 and group 2 have places left.

(1) If Alice joins group 1, the tutor refuses to accept Bob because they always talk.

(2) At least one of Bob and Claire cannot go to group 1, as they lead a chess group together that meets at the same time.

(3) Claire hates Alice and doesn't want to be in the same group.

(4) Alice wants to submit the solutions with either Bob or Claire and thus needs to be in a group with this person.

Model the above statements in propositional logic where the atomic propositions a (Alice), b (Bob), c (Claire) are assigned the value `true` if the corresponding person joins group 1, and `false` else.
Which persons join which group? Use a truth table to find out.

[1] `https://katalog.ub.uni-freiburg.de/opac/RDSIndex/Search?type0[]=ta&lookfor0[]=Principles+of+Model+Checking`

Exercise 2: Finite Automata — 9 Points

We practice working with formal languages and non-deterministic finite automata. This serves as preparation for a similar concept in the lecture, so-called transition systems*.*

Consider the following descriptions of two formal languages over an alphabet $\Sigma = \{a, b\}$.

(L_1) The language of all words such that the second-to-last letter is the letter a.

(L_2) The language of all words such that the first letter is equal to the last letter.

For these two languages, perform the following tasks.

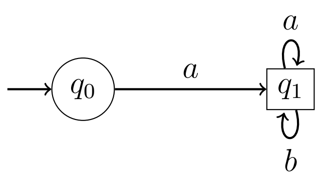
(a) Formally define these languages as sets of words.

Example: The language L_0 of all words that begin with the letter a can be defined as $L_0 = \{\, x_0 x_1 \ldots x_n \mid n \in \mathbb{N}_0 \text{ and } (\forall i \leq n \,.\, x_i \in \Sigma) \text{ and } x_0 = a \,\}$.

(b) For each of these languages, draw a finite automaton that recognizes the language. That is, draw an automaton A_1 that accepts a word w if and only if its second-to-last letter is an a. Similarly, draw an automaton A_2 that accepts a word w if and only if its first and last letters are equal.

Hint: There are automata with 4 states that recognize L_1 and L_2.

Example: An automaton A_0 that recognizes the language L_0 is shown below:



(c) Describe the automata from exercise *(b)* as a five-tuple.

Example: The automaton A_0 above can be described as $A_0 = (Q_0, \Sigma, \delta_0, Q_0^{\text{init}}, F_0)$ with the set of states $Q_0 = \{q_0, q_1\}$, the set of initial states $Q_0^{\text{init}} = \{q_0\}$, the set of accepting states $F_0 = \{q_1\}$ and the transition relation $\delta_0 = \{(q_0, a, q_1), (q_1, a, q_1), (q_1, b, q_1)\}$.