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Hand in until November 9th, 2022
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Tutorial for Cyber-Physical Systems - Discrete Models
Exercise Sheet 3

Exercise 1: Intersection of ω-regular languages 6 Points

The goal of this exercise is to create Büchi automata that accept the intersection of two ω-regular languages, as we are interested in the intersections in the lecture later on.

In the following exercise we are interested in the intersection of two ω-regular languages. The intersection is also an ω-regular language, so we can represent it in a Büchi automaton. We are asking you to first look at the language obtained by taking the intersection of the two given languages, and then construct the Büchi automaton for that language.

There is also an algorithm to intersect two Büchi automata. In fact, the exercise will help you to gain an intuition about the algorithm to intersect two Büchi automata. If you wish, you can try to figure out the algorithm on your own. **Be aware, however. One cannot simply take the construction of the finite automaton for the intersection you have seen in the exercise session (what will go wrong?).**

Given two ω-languages L_1 and L_2, a Büchi automaton A accepts the intersection of L_1 and L_2 if and only if $L_\omega(A) = L_1 \cap L_2$. This means a word $w \in \Sigma^\omega$ is accepted by A if and only if $w \in L_1$ and $w \in L_2$.

(a) Consider the following ω-languages over the alphabet $\Sigma = \{a, b, c\}$.

$$\begin{aligned} L_1 &= \{\, x_0 x_1 \ldots \mid \overset{\infty}{\forall} i \in \mathbb{N}_0 \,.\, x_i \neq a \} \\ &= \{\, x_0 x_1 \ldots \mid \exists j \in \mathbb{N}_0 \,.\, \forall i \in \mathbb{N}_0 \,.\, i > j \rightarrow x_i \neq a \} \\ L_2 &= \{\, x_0 x_1 \ldots \mid \overset{\infty}{\exists} i \in \mathbb{N}_0 \,.\, x_i = b \,\} \\ &= \{\, x_0 x_1 \ldots \mid \forall j \in \mathbb{N}_0 \,.\, \exists i \in \mathbb{N}_0 \,.\, i > j \wedge x_i = b \,\} \end{aligned}$$

Draw a Büchi automaton A that accepts the intersection of L_1 and L_2.

(b) Consider the following ω-languages over the alphabet $\Sigma = \{a, b\}$.

$$\begin{aligned} L_1 &= \{\, x_0 x_1 \ldots \mid \overset{\infty}{\exists} i \in \mathbb{N}_0 \,.\, x_i = a \,\} \\ &= \{\, x_0 x_1 \ldots \mid \forall j \in \mathbb{N}_0 \,.\, \exists i \in \mathbb{N}_0 \,.\, i > j \wedge x_i = a \,\} \\ L_2 &= \{\, x_0 x_1 \ldots \mid \overset{\infty}{\exists} i \in \mathbb{N}_0 \,.\, x_i = b \,\} \\ &= \{\, x_0 x_1 \ldots \mid \forall j \in \mathbb{N}_0 \,.\, \exists i \in \mathbb{N}_0 \,.\, i > j \wedge x_i = b \,\} \end{aligned}$$

Draw a Büchi automaton A that accepts the intersection of L_1 and L_2.

(c) Consider the following ω-languages over the alphabet $\Sigma = \{a, b\}$.

$$\begin{aligned}
L_1 &= \{\, x_0x_1 \ldots \mid \overset{\infty}{\forall} i \in \mathbb{N}_0 \,.\, x_i \neq a\} \\
&= \{\, x_0x_1 \ldots \mid \exists j \in \mathbb{N}_0 \,.\, \forall i \in \mathbb{N}_0 \,.\, i > j \to x_i \neq a\} \\
L_2 &= \{\, x_0x_1 \ldots \mid \forall i \in \mathbb{N}_0 \,.\, x_i = b \to x_{i+1} = a \,\}
\end{aligned}$$

Draw a Büchi automaton A that accepts the intersection of L_1 and L_2.

Exercise 2: Transition Systems 6 Points

The goal of this exercise is to understand the connection between mathematical notation and graphical representation of transition systems.

In the lecture, we went over the labeling of edges and the labeling of nodes rather quickly, so here is a refresher:

- If a transition from s to s' is labeled by the action α, then we have the triple (s, α, s') in the transition relation $\longrightarrow$.
- If the state s is labeled by the set of atomic propositions $\{A_1, \ldots, A_n\}$, then the image of s under the labeling function L is $\{A_1, \ldots, A_n\}$, formally $L(s) = \{A_1, \ldots, A_n\}$.
- Note that the set of atomic propositions $\{A_1, \ldots, A_n\}$ can be empty (which corresponds to $n = 0$).

(a) Let $\mathcal{T} = (S, \mathit{Act}, \to, S_0, AP, L)$ be a transition system with

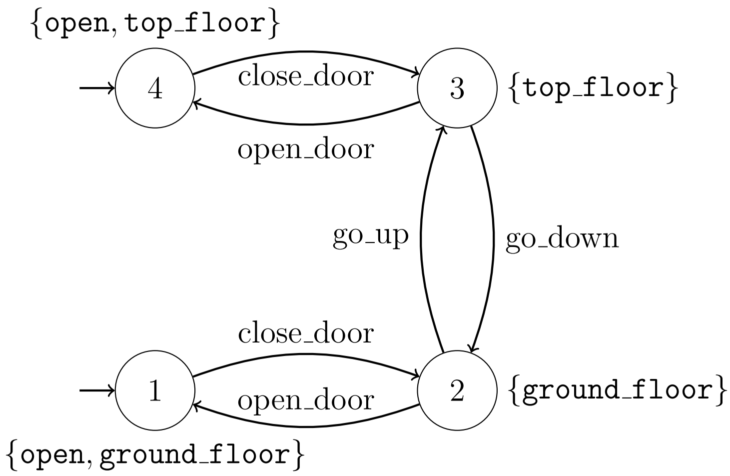
- the set of states $S = \{\texttt{locked}, \texttt{checking}, \texttt{opened}\}$,
- the set of action $\mathit{Act} = \{\texttt{insert\_ticket}, \texttt{unlock}, \texttt{enter}, \texttt{error}\}$,
- the transition relation

$$\begin{aligned}
\to\; = \{&(\texttt{locked}, \texttt{insert_ticket}, \texttt{checking}), (\texttt{checking}, \texttt{unlock}, \texttt{opened}), \\
&(\texttt{opened}, \texttt{enter}, \texttt{locked}), (\texttt{checking}, \texttt{error}, \texttt{locked})\},
\end{aligned}$$

- the initial states $S_0 = \{\texttt{locked}\}$,
- the set of atomic propositions $AP = \{\texttt{light\_red}, \texttt{light\_green}\}$,
- and the labeling function L with $L(\texttt{locked}) = \{\texttt{light\_red}\}$, $L(\texttt{checking}) = \emptyset$ and $L(\texttt{opened}) = \{\texttt{light\_green}\}$.

Draw this transition system. Can you see what cyber-physical system it models?

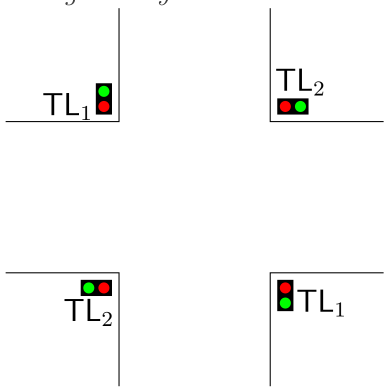
(b) The transition system shown below models an elevator. Give the corresponding mathematical definition, i.e., define the tuple $\mathcal{T} = (S, \mathit{Act}, \to, S_0, AP, L)$ that is described by the picture, in the style of (a).
In which states is the elevator door closed?



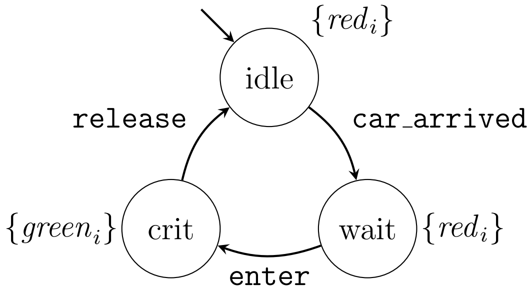
Exercise 3: Crossroads Traffic Lights 8 Points

The goal of this exercise is to practice the construction of interleaving and synchronization.

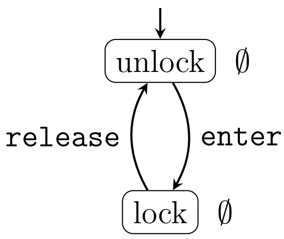
Consider the crossing of two roads with four traffic lights as depicted on the right. The two traffic lights labelled with TL_1 always show the same color, and likewise the two traffic lights labelled with TL_2 always show the same color. The traffic-lights are demand-driven and only switch to green when they have detected the presence of a car.



We model this system with two transition systems, TS_1 and TS_2, for the traffic lights, one for each direction of the crossing. To avoid car crashes, these transition systems are connected via an arbiter that ensures only one traffic light can be green at a time.



TS_i



Arbiter

(a) Draw the interleaving $TS_1 \,|||\, TS_2$ of the two transition systems for the traffic lights.

The traffic lights do not synchronize with each other. A configuration of the interleaving is labeled with all atomic propositions that hold in any of its local states.

(b) Draw the parallel composition $(TS_1 ||| TS_2) \parallel Arbiter$ of the three transition systems.

The traffic lights do not synchronize with each other, but they both synchronize with the arbiter. A configuration of the composed system is labeled with all atomic propositions that hold in any of its local states.

(c) Use the transition system from (*b*) to answer the following questions.

- Is the system safe, i.e., can the traffic lights for both streets be green at the same time? Explain using the atomic propositions.
- What other property would we expect from a traffic light? Why does it not hold here?