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Hand in until November 23rd, 2022
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Tutorial for Cyber-Physical Systems - Discrete Models

Exercise Sheet 5

Exercise 1: Synchronization — 2 Points

The goal of this exercise is to gain an understanding how the different parallel composition operators behave.

Given two transition systems $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ and $\mathcal{T}' = (S', Act', \rightarrow', S'_0, AP', L')$

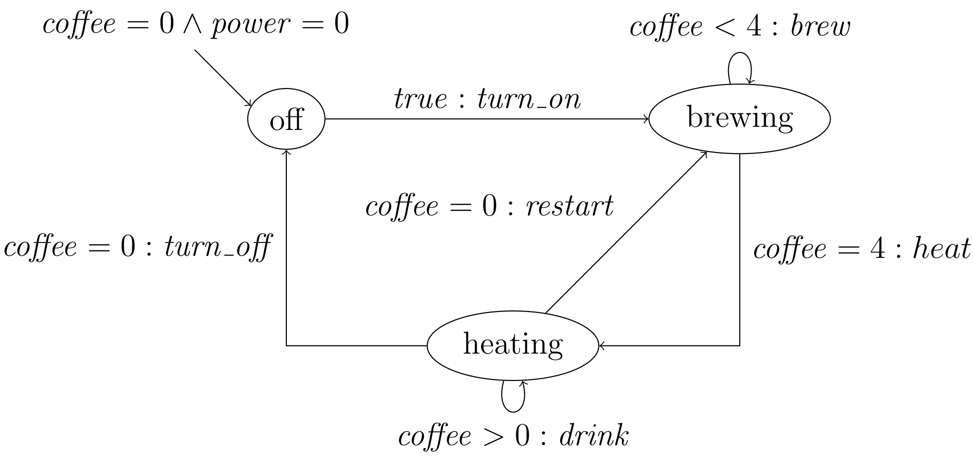
(a) Give a set Syn such that $\mathcal{T} \parallel \mathcal{T}'$ and $\mathcal{T} \parallel_{Syn} \mathcal{T}'$ are always equivalent.

(b) Give a set Syn such that $\mathcal{T} ||| \mathcal{T}'$ and $\mathcal{T} \parallel_{Syn} \mathcal{T}'$ are always equivalent.

Exercise 2: Coffee Machine and Transition System — 8 Points

The goal of this task is to provide some intuition on when the system described by a program graph satisfies given properties, by looking at the transition system.

The following program graph describes a simple coffee machine:



The effect of the operations is given by:

$$\begin{aligned}
\mathsf{Effect}(turn_on, \eta) &= \eta[power := 1] \\
\mathsf{Effect}(turn_off, \eta) &= \eta[power := 0] \\
\mathsf{Effect}(brew, \eta) &= \eta[coffee := coffee + 1] \\
\mathsf{Effect}(drink, \eta) &= \eta[coffee := coffee - 1] \\
\mathsf{Effect}(restart, \eta) &= \eta \\
\mathsf{Effect}(heat, \eta) &= \eta
\end{aligned}$$

(a) Draw the (reachable part of the) transition system corresponding to the program graph. Choose 3 transitions of the transition system, and justify their existence using the respective SOS-rule.

Use the SOS-rules to argue why the following transitions are *not* part of the transition system:

$$\langle \mathsf{off}, \{\mathit{coffee} \mapsto 0, \mathit{power} \mapsto 0\}\rangle \xrightarrow{\mathit{heat}} \langle \mathsf{heating}, \{\mathit{coffee} \mapsto 0, \mathit{power} \mapsto 0\}\rangle$$
$$\langle \mathsf{brewing}, \{\mathit{coffee} \mapsto 4, \mathit{power} \mapsto 1\}\rangle \xrightarrow{\mathit{brew}} \langle \mathsf{brewing}, \{\mathit{coffee} \mapsto 5, \mathit{power} \mapsto 1\}\rangle$$

(b) Use the transition system to explain which of the following properties are true for every execution of the coffee machine.

(i) If the machine is turned off ($\mathit{power} = 0$), it contains no coffee ($\mathit{coffee} = 0$).

(ii) If there are two cups of coffee ($\mathit{coffee} = 2$), there are either three or four cups of coffee in the next step ($\mathit{coffee} = 3$, $\mathit{coffee} = 4$).

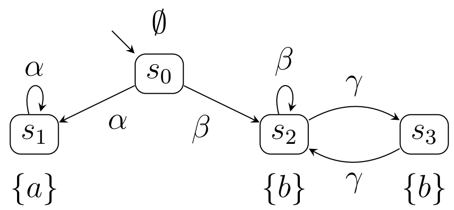
(iii) There are always at most four cups of coffee ($\mathit{coffee} \leq 4$).

(iv) The coffee machine will be turned off (i.e., in location *off*) infinitely often.

(v) If there is no coffee ($\mathit{coffee} = 0$), there will be coffee after at most three steps.

Exercise 3: Executions, Paths and Traces 5+2 Points

Consider the following transition system with the set of atomic propositions $AP = \{a, b\}$.



Solve the following tasks.

(a) Give examples that illustrate the difference between the different notions of executions and execution fragments. Therefore give the following execution fragments of the given transition system:

- An execution fragment that is neither initial nor maximal
- An initial execution fragment that is not maximal
- A maximal execution fragment that is not initial
- An initial and maximal execution fragment (i.e. an execution)

(b) How many executions does the transition system have? Explain your answer.

(c) Provide a path of the transition system. How many are there in total?

(d) How many traces does the transition system have? Provide all of them.

(e) **Bonus:** Is it possible to have a transition system with infinitely many executions and only finitely many paths? If yes, provide such a transition system, otherwise explain why this is not possible.