

## Tutorial for Cyber-Physical Systems - Discrete Models

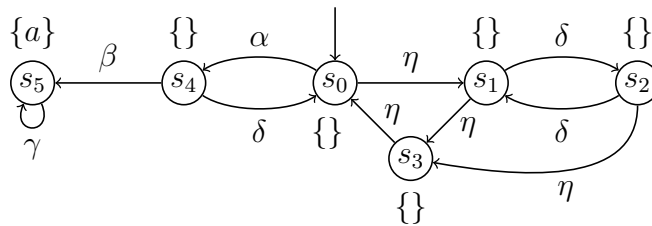
### Exercise Sheet 11

#### Exercise 1: Satisfaction under Fairness Assumptions

12 Points

*The goal of this task is to train your ability to identify fair and unfair traces of a given transition system, in order to reason about properties of a system under given fairness assumptions.*

Consider the following transition system:



For the fairness assumptions given in (a)–(h), perform the following tasks.

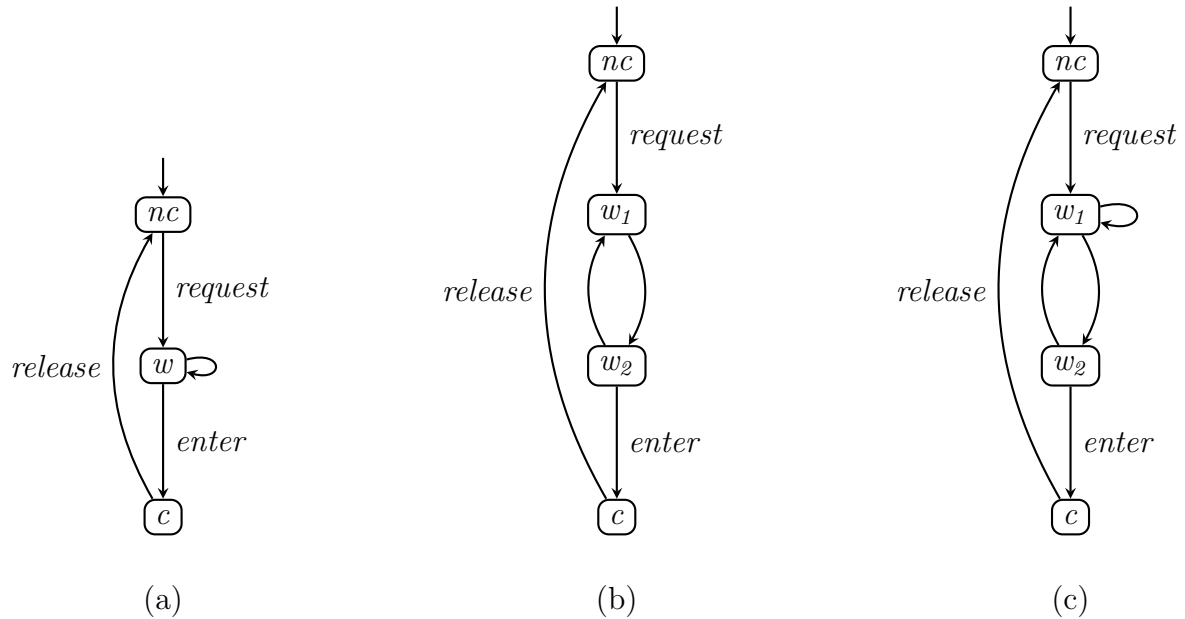
- (i) For each of the fairness assumptions below, give an execution that fulfills the fairness assumption (a fair execution) and an execution that violates the fairness assumption (an unfair execution).
- (ii) A system satisfies a property  $P$  under a given fairness assumption, if all fair traces (i.e., traces corresponding to fair executions) satisfy property  $P$ .  
 Under which of the following fairness assumptions does the system satisfy the property “eventually  $a$ ”? Justify your answer.
  - (a) unconditional fairness for  $A = \{\gamma\}$
  - (b) unconditional fairness for  $A_1 = \{\alpha\}$  and for  $A_2 = \{\gamma\}$
  - (c) unconditional fairness for  $A = \{\alpha, \gamma\}$
  - (d) strong fairness for  $A = \{\beta\}$
  - (e) strong fairness for  $A_1 = \{\alpha\}$  and for  $A_2 = \{\beta\}$
  - (f) strong fairness for  $A_1 = \{\alpha\}$  and for  $A_2 = \{\beta\}$  and for  $A_3 = \{\eta\}$
  - (g) weak fairness for  $A = \{\eta\}$
  - (h) weak fairness for  $A_1 = \{\alpha\}$  and for  $A_2 = \{\beta\}$  and for  $A_3 = \{\eta\}$

**Exercise 2: Fairness Assumptions**

6 Points

For each of the following three systems (each consisting of one single process) give the weakest fairness assumption on action *enter* to ensure non-starvation. Non-starvation means that a process that has requested will eventually enter its critical section.

Give an informal explanation for your answers. Explain both why your chosen fairness assumption is *sufficient* to ensure non-starvation, and why it is the *weakest* possible assumption.



**Exercise 3\*: Closure Properties of LT Properties**

6 Bonus Points

The goal of this task is to understand the effect of set operations on liveness and safety properties.

Let  $P$  and  $P'$  be liveness properties over  $AP$ . Prove or disprove the following claims:

- (a)  $P \cup P'$  is a liveness property.
- (b)  $P \cap P'$  is a liveness property.

Perform the same tasks for safety properties.