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# Tutorial for Cyber-Physical Systems - Discrete Models
## Exercise Sheet 12

**Exercise 1*: Lecture Evaluation** — 1 Bonus Point

Complete the lecture evaluation.

**Exercise 2: LTL Properties** — 12 Points

Given the following LTL properties over $AP = \{a, b, c\}$:

$\varphi_1 = a \wedge \bigcirc b$ $\qquad$ $\varphi_3 = \neg(a \mathbin{\mathsf{U}} \Box b)$ $\qquad$ $\varphi_5 = \Diamond \Box a$

$\varphi_2 = a \mathbin{\mathsf{U}} b$ $\qquad$ $\varphi_4 = (\Diamond c) \mathbin{\mathsf{U}} \Box a$ $\qquad$ $\varphi_6 = \Box \Diamond c$

For each of the LTL properties $\varphi_i$ complete the following tasks:

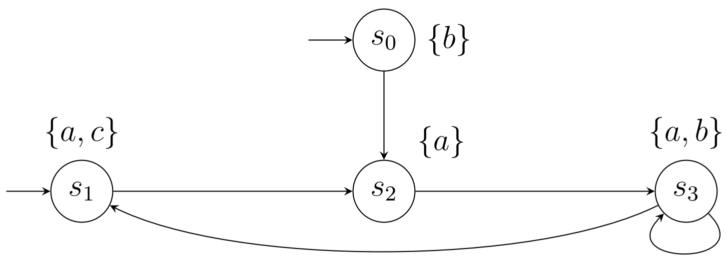
(a) Give a trace $\tau \in (2^{AP})^\omega$ that satisfies $\varphi_i$.

(b) Give a trace $\tau \in (2^{AP})^\omega$ that violates $\varphi_i$.

(c) State whether or not the transition system below satisfies $\varphi_i$.

(d) Formalize $Words(\varphi_i)$ (i.e. the set of all traces satisfying $\varphi_i$) using set comprehension.

For example for $\varphi = \Diamond a$ we can formalize $Words(\varphi) = \{A_0 A_1 \cdots \mid \exists i.\ a \in A_i\}$.




**Exercise 3: Stating properties in LTL** — 3 Points + 2 Bonus Points

Suppose we have two users, *Betsy* and *Peter*, and a single printer device. Both users perform several tasks, and every now and then they want to print their results on the printer. Since there is only a single printer, only one user can print a job at a time. Suppose we have the following atomic propositions for *Peter* at our disposal:

| | |
|---|---|
| *Peter.request* | indicates that *Peter* requests usage of the printer. |
| *Peter.use* | indicates that *Peter* uses the printer. |
| *Peter.release* | indicates that *Peter* releases the printer. |

For *Betsy*, analogous predicates are defined. Specify in LTL the following properties:

(a) Mutual exclusion, i.e., only one user at a time can use the printer.

(b) Finite time of usage, i.e., a user can print only for a finite amount of time.

(c) Absence of individual starvation, i.e., if a user wants to print something, the user is eventually able to do so.

(d) **Bonus:** Absence of blocking, i.e., if a user requests access to the printer, the user does not request forever.

(e) **Bonus:** Alternating access, i.e., users must strictly alternate in printing.

**Exercise 4: Equivalence of LTL formulas** 8 Points + 2 Bonus Points

Consider the following claims about equivalences of LTL formulas.
Provide a counterexample (i.e. a transition system that satisfies one of the properties and violates the other) if an equivalence does not hold.

(a) $\square a \land \bigcirc \lozenge a \overset{?}{\equiv} \square a$

(b) $\lozenge a \land \bigcirc \square a \overset{?}{\equiv} \lozenge a$

(c) $\square a \rightarrow \lozenge b \overset{?}{\equiv} a \ \mathsf{U} \ (b \lor \neg a)$

(d) $a \ \mathsf{U} \ \mathit{false} \overset{?}{\equiv} \square a$

(e) $\square \bigcirc b \overset{?}{\equiv} \square b$

**Bonus:** If an equivalence holds, give a proof.