

# *Software Design, Modelling and Analysis in UML*

## *Lecture 08: Class Diagrams III*

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Prof. Dr. Andreas Podelski, **Dr. Bernd Westphal**

Albert-Ludwigs-Universität Freiburg, Germany

## Contents & Goals

### **Last Lectures:**

- Started to discuss “associations”, the general case.

### **This Lecture:**

- Educational Objectives:** Capabilities for following tasks/questions.
  - Cont'd: Please explain this class diagram with associations.
  - When is a class diagram a good class diagram?
  - What are purposes of modelling guidelines? (Example?)
  - Discuss the style of this class diagram.

### **Content:**

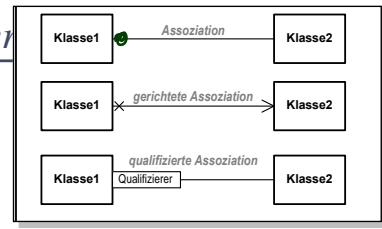
- Recall association semantics and effect on OCL.
- Treat “the rest”.
- Where do we put OCL constraints?
- Modelling guidelines, in particular for class diagrams (following [Ambler, 2005])
- Examples: modelling games (made-up and real-world examples)

## Recall: Associations and OCL

### Recall: What Do We (Have to) Cover

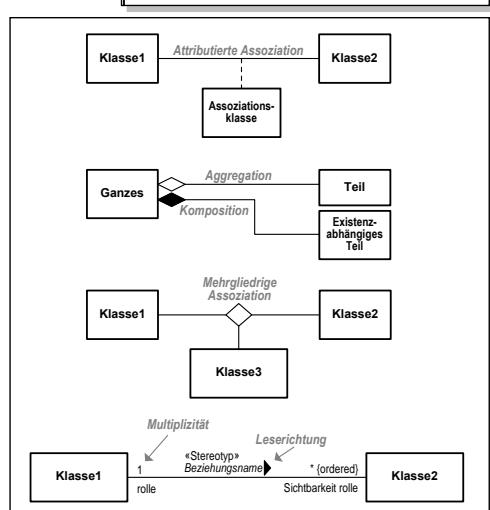
An **association** has

- a **name**,
- a **reading direction**, and
- at least two **ends**.



Each **end** has

- a **role name**,
- a **multiplicity**,
- a set of **properties**, such as **unique**, **ordered**, etc.
- a **qualifier**,
- a **visibility**,
- a **navigability**,
- an **ownership** (not in pictures),
- and possibly a **diamond**.

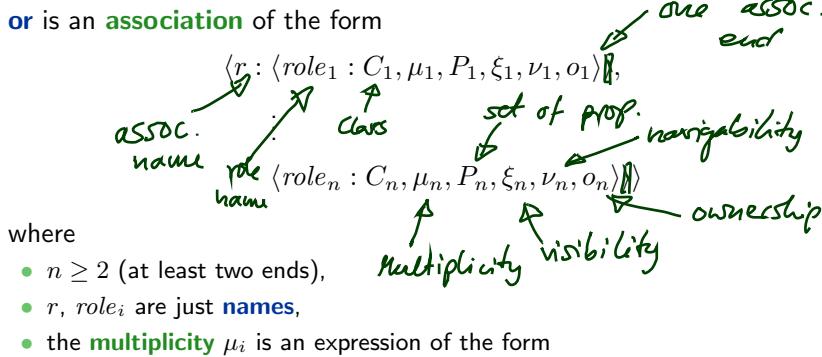


**Wanted:** places in the signature to represent the information from the picture.

## Recall: (Temporarily) Extend Signature: Associations

**Only** for the course of Lectures 07/08 we assume that each attribute in  $V$

- **either** is  $\langle v : \tau, \xi, \text{expr}_0, P_v \rangle$  with  $\tau \in \mathcal{T}$  (as before),
- **or** is an **association** of the form



$$\mu ::= * \mid N \mid N..M \mid N..* \mid \mu, \mu \quad (N, M \in \mathbb{N})$$

- $P_i$  is a set of **properties** (as before),
- $\xi \in \{+, -, \#, \sim\}$  (as before),
- $\nu_i \in \{\times, -, >\}$  is the **navigability**,
- $o_i \in \mathbb{B}$  is the **ownership**.

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## Recall: Associations in General

**Recall:** We consider associations of the following form:

$$\langle r : \langle role_1 : C_1, \mu_1, P_1, \xi_1, \nu_1, o_1 \rangle \rangle, \dots, \langle role_n : C_n, \mu_n, P_n, \xi_n, \nu_n, o_n \rangle \rangle \rangle$$

Only these parts are relevant for extended system states:

$$\langle r : \langle role_1 : C_1, -, P_1, -, -, - \rangle, \dots, \langle role_n : C_n, -, P_n, -, -, - \rangle \rangle$$

(recall: we assume  $P_1 = P_n = \{\text{unique}\}$ ).

The UML standard thinks of associations as **n-ary relations** which “**live on their own**” in a system state.

That is, **links** (= association instances)

- **do not** belong (in general) to certain objects (in contrast to pointers, e.g.)
- are “first-class citizens” **next to objects**,
- are (in general) **not** directed (in contrast to pointers).

## Recall: Links in System States

$$\langle r : \langle role_1 : C_1, \_, P_1, \_, \_, \_ \rangle, \dots, \langle role_n : C_n, \_, P_n, \_, \_, \_ \rangle \rangle$$

**Only** for the course of this lecture we change the definition of system states:

**Definition.** Let  $\mathcal{D}$  be a structure of the (extended) signature  $\mathcal{S} = (\mathcal{T}, \mathcal{C}, V, atr)$ .

A **system state** of  $\mathcal{S}$  wrt.  $\mathcal{D}$  is a pair  $(\sigma, \lambda)$  consisting of

- a type-consistent mapping  $\sigma : \mathcal{D}(\mathcal{C}) \rightarrow (atr(\mathcal{C}) \rightarrow \mathcal{D}(\mathcal{T}))$
- a mapping  $\lambda$  which assigns each association  $\langle r : \langle role_1 : C_1, \dots, role_n : C_n \rangle \rangle \in V$  a relation  $\lambda(r) \subseteq \mathcal{D}(C_1) \times \dots \times \mathcal{D}(C_n)$

(i.e. a set of type-consistent  $n$ -tuples of identities).

Q: Should it better be

$$\lambda(r) \subseteq \text{dom}(\sigma)^n ?$$

(i.e. only alive objects participate in links)

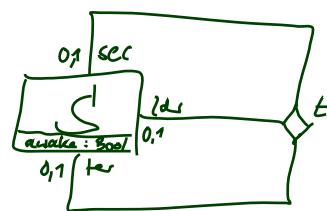
A: choice of lecture: NO



(complete)

↳ p in 2; D is a dangling reference,  
5c is maybe no longer alive

## Example



$\langle t : \langle ldr : S^{'}, \dots \rangle,$   
 $\langle sec : S^{'}, \dots \rangle,$   
 $\langle ter : S^{'}, \dots \rangle \rangle$

$\sigma_1 : \{ 1_S \mapsto \{ \text{aw} \mapsto 1 \}, 2_S \mapsto \{ \text{aw} \mapsto 0 \}, 3_S \mapsto \{ \text{aw} \mapsto 1 \}, 27_S \mapsto \{ \text{aw} \mapsto 1 \} \}$

$\lambda_1 : t \mapsto \{ (1_S, 3_S, 2_S),$   
 $(1_S, 27_S, 3_S),$   
 $(2_S, 5_S, 6_S), \quad // \text{students } 5_S, 6_S \text{ left university} \quad (*)$   
 $(3_S, 3_S, 3_S) \} \quad // \text{one student playing all three roles}$

If (\*) is not desired, add:  
 context  $S$  inv:  $ldr \neq sec$  and  $sec \neq ter$

## Associations and OCL

## OCL and Associations: Syntax

**Recall:** OCL syntax as introduced in Lecture 03, interesting part:

$expr ::= \dots   r_1(expr_1) : \tau_C \rightarrow \tau_D$	$r_1 : D_{0,1} \in atr(C)$
$  r_2(expr_1) : \tau_C \rightarrow Set(\tau_D)$	$r_2 : D_* \in atr(C)$

**Now becomes**

$expr ::= \dots   role(expr_1) : \tau_C \rightarrow \tau_D$	$\mu = 0..1 \text{ or } \mu = 1$
$  role(expr_1) : \tau_C \rightarrow Set(\tau_D)$	otherwise

if

$\langle r : \dots, \langle role : D, \mu, \_, \_, \_, \_ \rangle, \dots, \langle role' : C, \_, \_, \_, \_, \_ \rangle, \dots \rangle \in V \text{ or}$   
 $\langle r : \dots, \langle role' : C, \_, \_, \_, \_, \_ \rangle, \dots, \langle role : D, \mu, \_, \_, \_, \_ \rangle, \dots \rangle \in V, role \neq role'.$

**Note:**

- Association name as such doesn't occur in OCL syntax, role names do.
- $expr_1$  has to denote an object of a class which "participates" in the association.

## OCL and Associations Syntax: Example

$expr ::= \dots   \underline{role(expr_1)} : \tau_C \rightarrow \tau_D$	$\mu = 0..1 \text{ or } \mu = 1$
$  role(expr_1) : \tau_C \rightarrow Set(\tau_D)$	otherwise

if

$\langle r : \dots, \langle role : D, \mu, \_, \_, \_, \_ \rangle, \dots, \langle role' : C, \_, \_, \_, \_, \_ \rangle, \dots \rangle \in V \text{ or}$   
 $\langle r : \dots, \langle role' : C, \_, \_, \_, \_, \_ \rangle, \dots, \langle role : D, \mu, \_, \_, \_, \_ \rangle, \dots \rangle \in V, role \neq role'.$

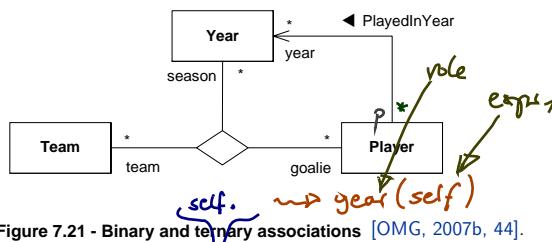


Figure 7.21 - Binary and ternary associations [OMG, 2007b, 44].

- ① context Player inv: size( year ) > 0
- ② NOT: context Player inv: size( p ) > 0
- ③ context Player inv: size( season ) > 0
- ④ NOT: context Player inv: size( goalie ) > 0

## OCL and Associations: Semantics

**Recall:** (Lecture 03)

Assume  $expr_1 : \tau_C$  for some  $C \in \mathcal{C}$ . Set  $u_1 := I[\![expr_1]\!](\sigma, \beta) \in \mathcal{D}(\tau_C)$ .

- $I[\![r_1(expr_1)]\!](\sigma, \beta) := \begin{cases} u & , \text{ if } u_1 \in \text{dom}(\sigma) \text{ and } \sigma(u_1)(r_1) = \{u\} \\ \perp & , \text{ otherwise} \end{cases}$

- $I[\![r_2(expr_1)]\!](\sigma, \beta) := \begin{cases} \sigma(u_1)(r_2) & , \text{ if } u_1 \in \text{dom}(\sigma) \\ \perp & , \text{ otherwise} \end{cases}$

**Now needed:**

$$I[\![role(expr_1)]\!]((\underline{\sigma}, \lambda), \beta)$$

- We cannot simply write  $\sigma(u)(role)$ .

**Recall:**  $role$  is (**for the moment**) not an attribute of object  $u$  (not in  $atr(C)$ ).

- What we have is  $\lambda(r)$  (with  $r$ , not with  $role!$ ) — but it yields a set of  $n$ -tuples, of which **some** relate  $u$  and other some instances of  $D$ .
- $role$  denotes the position of the  $D$ 's in the tuples constituting the value of  $r$ .

## OCL and Associations: Semantics Cont'd

**Assume**  $expr_1 : \tau_C$  for some  $C \in \mathcal{C}$ . Set  $u_1 := I[\![expr_1]\!](\sigma, \lambda, \beta) \in \mathcal{D}(\tau_C)$ .

- $I[\![role(expr_1)]\!]((\sigma, \lambda), \beta) := \begin{cases} u & , \text{ if } u_1 \in \text{dom}(\sigma) \text{ and } L(role)(u_1, \lambda) = \{u\} \\ \perp & , \text{ otherwise} \end{cases}$

- $I[\![role(expr_1)]\!]((\sigma, \lambda), \beta) := \begin{cases} L(role)(u_1, \lambda) & , \text{ if } u_1 \in \text{dom}(\sigma) \\ \perp & , \text{ otherwise} \end{cases}$

where

"database lookup"      *assumes r is uniquely determined by role*  
 $L(role)(u, \lambda) = \{(u_1, \dots, u_n) \in \lambda(r) \mid u \in \{u_1, \dots, u_n\}\}_{\downarrow i}$       *project onto its comp.*  
*select those tuples where u occurs at some position*

if

$$\langle r : \dots \langle role_1 : \_, \_, \_, \_, \_, \_ \rangle, \dots \langle role_n : \_, \_, \_, \_, \_, \_ \rangle, \dots \rangle, role = \underline{role_i} \rangle.$$

Given a set of  $n$ -tuples  $A$ ,  $A \downarrow i$  denotes the element-wise projection onto the  $i$ -th component.



$$\sigma = \{ 1 \mapsto \{ p \mapsto \{ 2 \} \}, \\ n \mapsto \{ 3, 4 \}, \\ 2 \mapsto \{ p \mapsto \emptyset, n \mapsto \emptyset \}$$

### OCL and Associations Example

$I[\text{role(expr}_1)]((\sigma, \lambda), \beta) := \begin{cases} L(\text{role})(u_1, \lambda) & , \text{ if } u_1 \in \text{dom}(\sigma) \\ \perp & , \text{ otherwise} \end{cases}$ $L(\text{role})(u, \lambda) = \{(u_1, \dots, u_n) \in \lambda(r) \mid u \in \{u_1, \dots, u_n\}\} \downarrow i$
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$$\sigma = \{ 1_C \mapsto \emptyset, 3_D \mapsto \{ x \mapsto 1 \}, 7_D \mapsto \{ x \mapsto 2 \} \}$$

$$\lambda = \{ A.C \rightarrow D \mapsto \{ (1_C, 3_D), (1_C, 7_D) \} \}$$

$$\begin{aligned}
 I[\text{self}.n]((\sigma, \lambda), \{ \text{self} \mapsto 1_C \}) &= I[n(\text{self})](\sigma, \lambda, \{ \text{self} \mapsto 1_C \}) \\
 &= \sqcup(n)(I[\text{self}]((\sigma, \lambda), \{ \text{self} \mapsto 1_C \}), \lambda) \\
 &= \sqcup(n)(1_C, \lambda) \\
 &= (\{ (1_C, 3_D), (1_C, 7_D) \}) \downarrow 2 \\
 &= \{ 3_D, 7_D \}
 \end{aligned}$$

## *Associations: The Rest*

### The Rest

**Recapitulation:** Consider the following association:

$$\langle r : \langle role_1 : C_1, \mu_1, P_1, \xi_1, \nu_1, o_1 \rangle \rangle, \dots, \langle role_n : C_n, \mu_n, P_n, \xi_n, \nu_n, o_n \rangle \rangle \rangle$$

- **Association name**  $r$  and **role names/types**  $role_i/C_i$  induce extended system states  $\lambda$ . ✓
- **Multiplicity**  $\mu$  is considered in OCL syntax.
- **Visibility**  $\xi$ /**Navigability**  $\nu$ : well-typedness.



**Now the rest:**

- **Multiplicity**  $\mu$ : we propose to view them as constraints.
- **Properties**  $P_i$ : even more typing.
- **Ownership**  $o$ : getting closer to pointers/references.
- **Diamonds**: exercise.

## *References*

## **References**

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- [Ambler, 2005] Ambler, S. W. (2005). *The Elements of UML 2.0 Style*. Cambridge University Press.
- [OMG, 2007a] OMG (2007a). Unified modeling language: Infrastructure, version 2.1.2. Technical Report formal/07-11-04.
- [OMG, 2007b] OMG (2007b). Unified modeling language: Superstructure, version 2.1.2. Technical Report formal/07-11-02.